
# Fair reward allocations for energy communities: bridging the gap between game theory and practical allocations

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# Main topics of this presentation

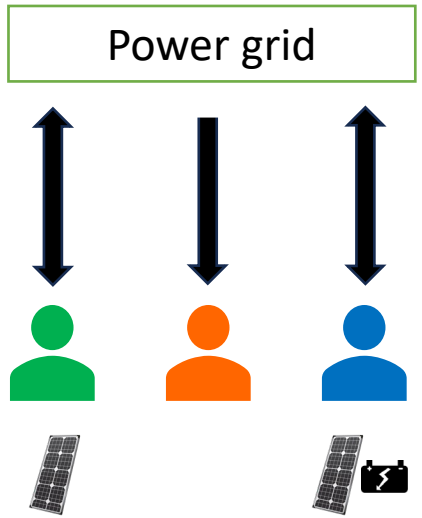
- **Optimal sizing and operation of Energy Communities**
- **The added value of a cooperative community**
- **Cooperative Game Theory: fairness and stability of ECs**
- **Simplified approaches to reward allocation (work in progress)**

# 1) How to quantify the total benefit of an Energy Community?

## Non-Cooperative (NC)

(no EC, each prosumer defines his own design and operation)

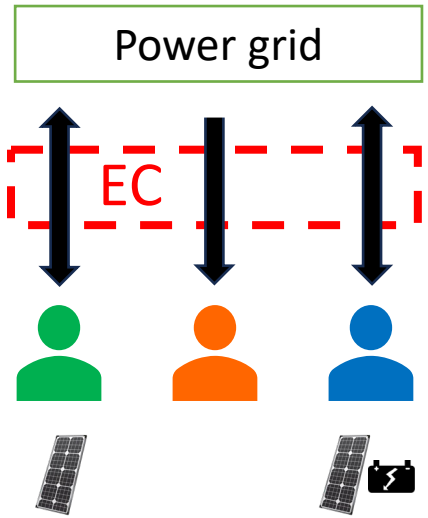
$$NPC_j = \sum_{y \in Y} \frac{C^P_{j,y} + I^U_{j,y} + RC^U_{j,y} - RV^U_{j,y} - R^P_{j,y}}{(1+d)^y}$$




No reward, no shared energy

## Aggregated Non-Cooperative (ANC)

(EC, same design and oper. of NC)




**Reward** for <u>accidental</u> $E^{SHARED}$

EC creation

EC optimization

**Added value of CO w.r.t. NC**

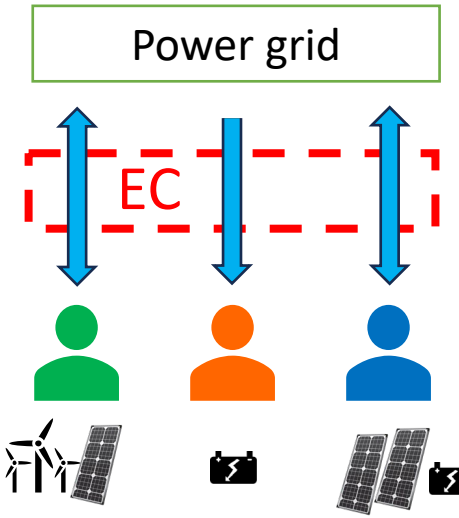
$$v(EC) = NPV^{CO} - NPV^{NC}$$

## Cooperative EC (CO)

(EC + portfolio **optimization**)

$$NPV^{CO} = \max \; R^{SH} - \sum_j NPC_j$$

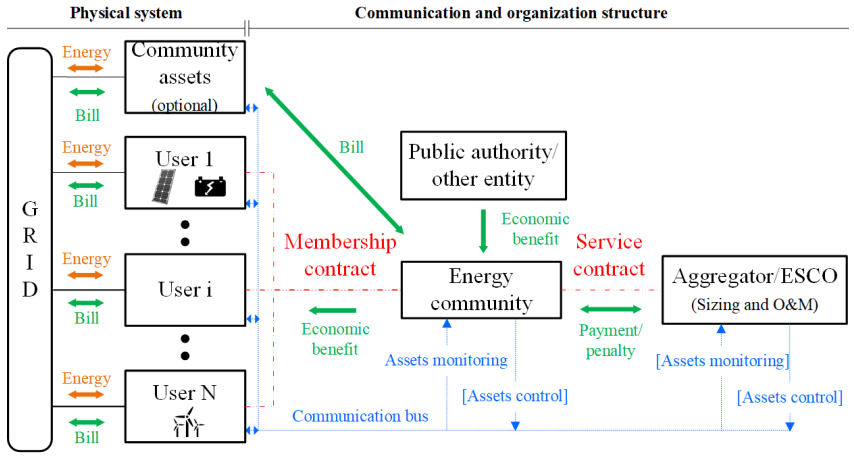
$$R^{SH} = \sum_y \frac{\sum_t m^T_t \pi^{SH}_t E^{SH}_t}{(1+d)^y}$$




aggregator

Reward for $E^{SHARED}$

# A tool for the optimization of ECs




Flexible number of users and components (batteries, renewables, loads) for each user

Linear Programming formulation (MILP)

EnergyCommunity.jl

NC, ANC, **CO**

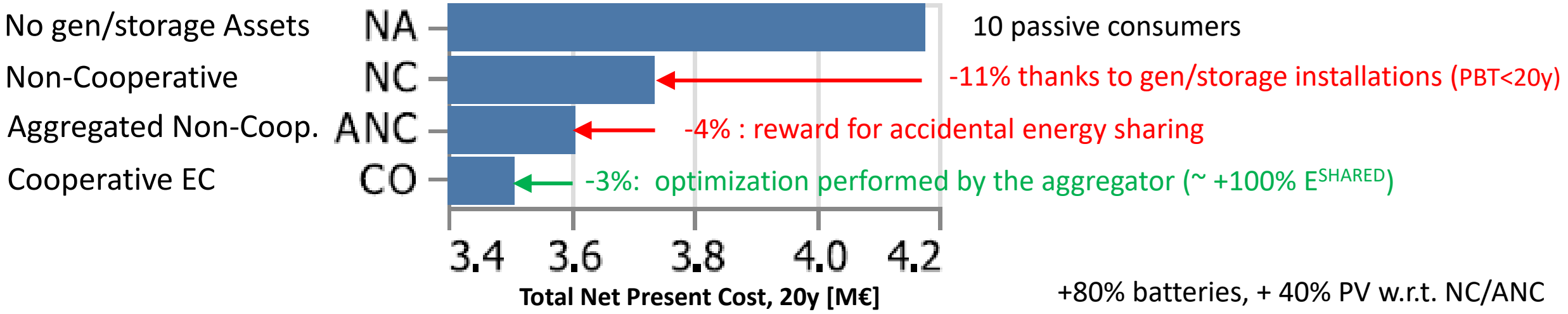
julia

**Optimal design and operation of single users (NC/ANC) and of the community (CO)**

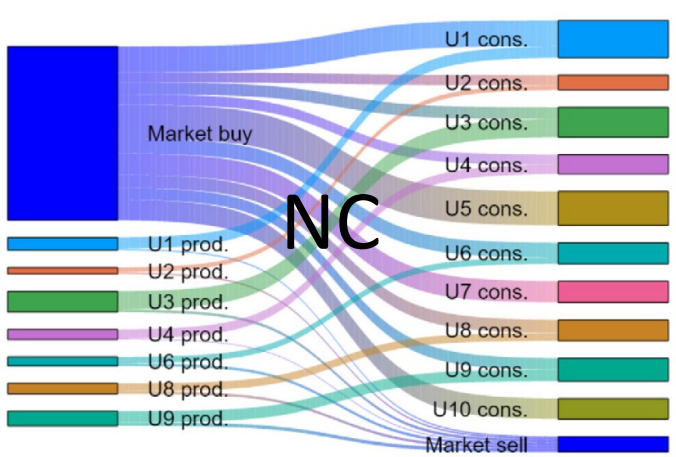
- Sizing of assets (gen. & storage)
- Operation of dispatchable assets
- Energy and cash flows
- Settlement

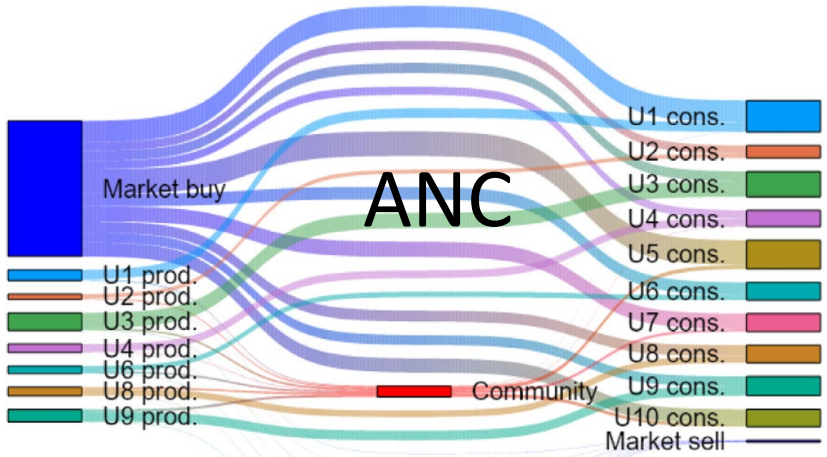
T.Ferrucci, D.Fioriti, D.Poli, «Optimising Energy Communities: multi-scenario analysis across prosumer penetration levels and community scales», *EEEIC 2025*, Crete, July 2025.

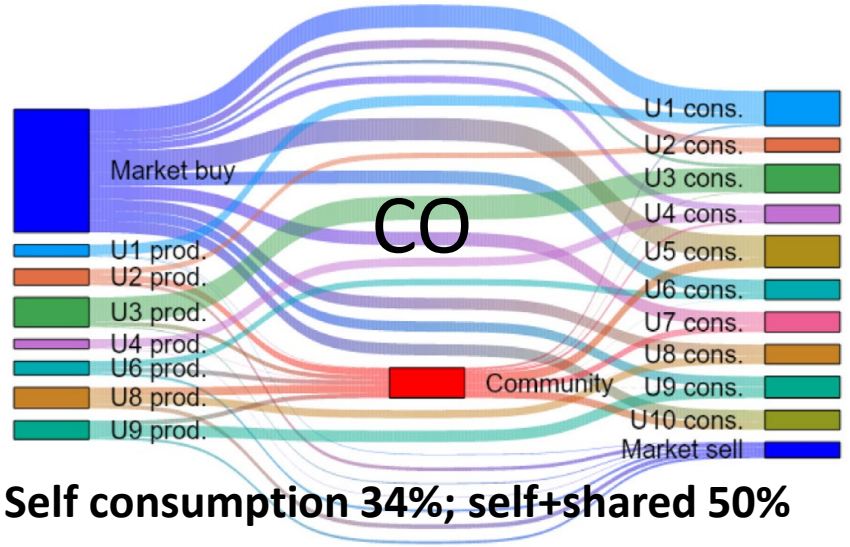
# A case study (7 prosumers+3 consumers; residential/commercial LV points)




+80% batteries, + 40% PV w.r.t. NC/ANC
Optimal operation **of batteries**



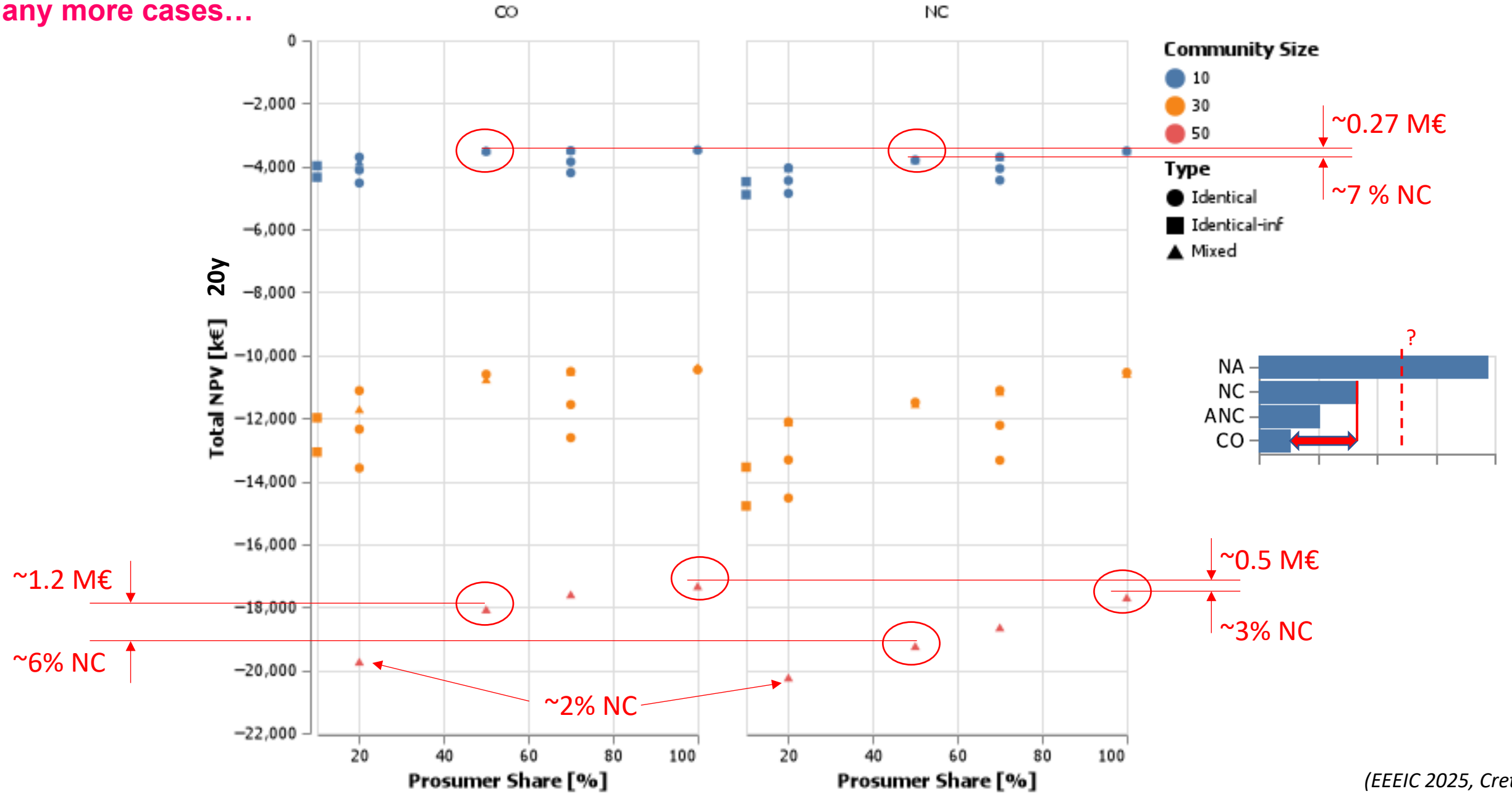




**Self consumed generation: 29% of total energy demand; self+shared 37%**

**Self consumption 34%; self+shared 50%**

D.Fioriti, A.Frangioni, D.Poli, «Optimal sizing of energy communities with fair revenue sharing and exit clauses: value, role and business model of aggregators and users», *Applied Energy*, 299 (2021)

# Many more cases…




*(EEEIC 2025, Crete)*

# 2) How to share the total benefit of the community among its members?

**Rules of allocation should be set so that each member of the EC properly benefits from participating and staying in the community**

1. **Fairness**: Members contributing more, should receive more
2. **Stability**: Any possible subgroup should receive more in the EC than creating its own coalition

$$\nexists\, J \subset EC \mid \sum_{j \in J} (NPV_j^{CO} - NPV_j^{NC}) < v(J) = NPV^{CO}(J) - NPV^{NC}(J)$$

*benefit received in the EC by the j-th member ($DNPV_j$)*

*total benefit the subgroup J would receive by creating its own community*

**"Reward allocation ∈ CORE"**

The set of stable reward allocations

**Cooperative Game Theory? … maybe with simplifications!**

# Cooperative Game Theory: highly mathematical and robust, but very complex!

## Shapley Value allocation

The «holy grail» of fairness, unique but often *unstable*
Computationally very challenging!

$$DNPV_j^{SV} = \frac{1}{|EC|} \sum_{J \subseteq EC} \binom{|EC|-1}{|J|}^{-1} [v(J) - v(J/j)]$$

marginal contribution provided by member *j* to the sub-coalition *J*

## Least Core allocation(s)

Strictly stable allocation (provided that $\vartheta > 0$)
*But not necessarily unique*

$$\max \theta$$

surplus of the least profitable coalition

$$\sum_{j \in J} DNPV_j^{LC} - v(J) \geq \theta \, \forall J \subset EC$$

profitability ("**surplus**") of the members of generic subgroup *J* to remain in the EC

## Nucleolus allocation

Unique, usually fair and stable, *but very hard to compute*

$$\max \theta$$

$$\sum_{j \in \hat{J}} DNPV_j^{Nuc} - v(\hat{J}) \geq \theta \quad \forall \hat{J} \notin \Gamma$$

$$\sum_{j \in \hat{J}} DNPV_j^{Nuc} - v(\hat{J}) \geq \theta_J \quad \forall \hat{J} \in \Gamma$$

## Variance Least Core allocation

Stable and pretty fair, easier to compute
Allocation inside the Least Core that is closest to uniform distribution

$$min \sum_{j \in J} (DNPV_j^{VLC} - \frac{1}{|J|} \sum_{j \in J} DNPV_j^{VLC})^2$$

uniform allocation

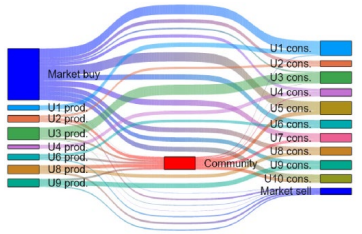
$$\sum_{j \in J} DNPV_j^{VLC} - v(J) \geq \hat{\theta} \, \forall J \subset EC$$

Least Core

D.Fioriti, G.Bigi, A.Frangioni, M.Passacantando, D.Poli, «Fair Least Core: efficient, stable and unique game-theoretic reward allocation in Energy Communities by row-generation», *IEEE Transactions on Energy Markets, Policy, and Regulation*, vol.3 (2), June 2025.

# Computational aspects

**Community features** → **Optimal design of the system** → **Cooperative game theory** → **Simplification of reward schemes** →




EnergyCommunity.jl

julia

TheoryOfGames.jl

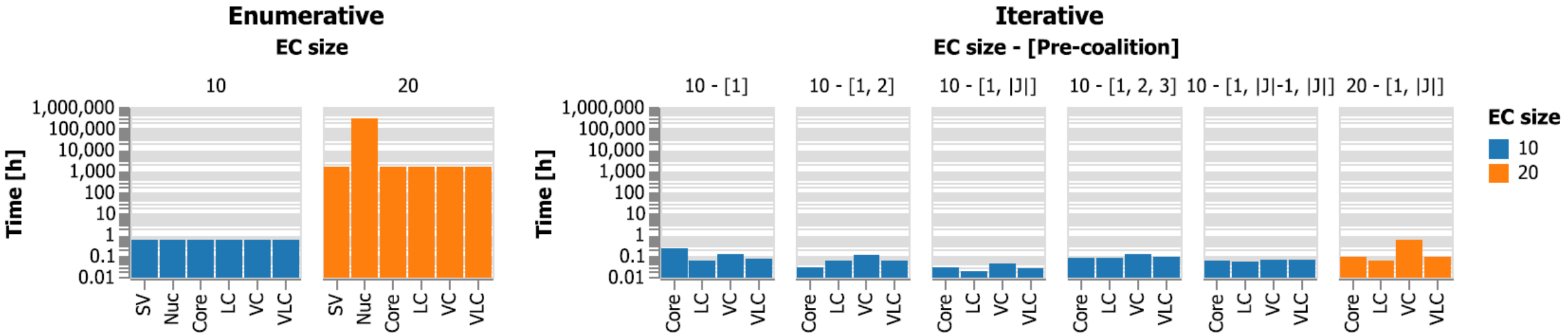
julia

Number of $v(J)$ calculations to perform: $2^{|EC|}$

Only a small fraction of the constraints are going to be binding →

**Row-generation approaches**

- Iterative calculation
- Master + Separation Problem
- SP: from combinatorial to MILP
  → comput. time = k * |EC|



# A 10-members EC

**Optimal design of the system**

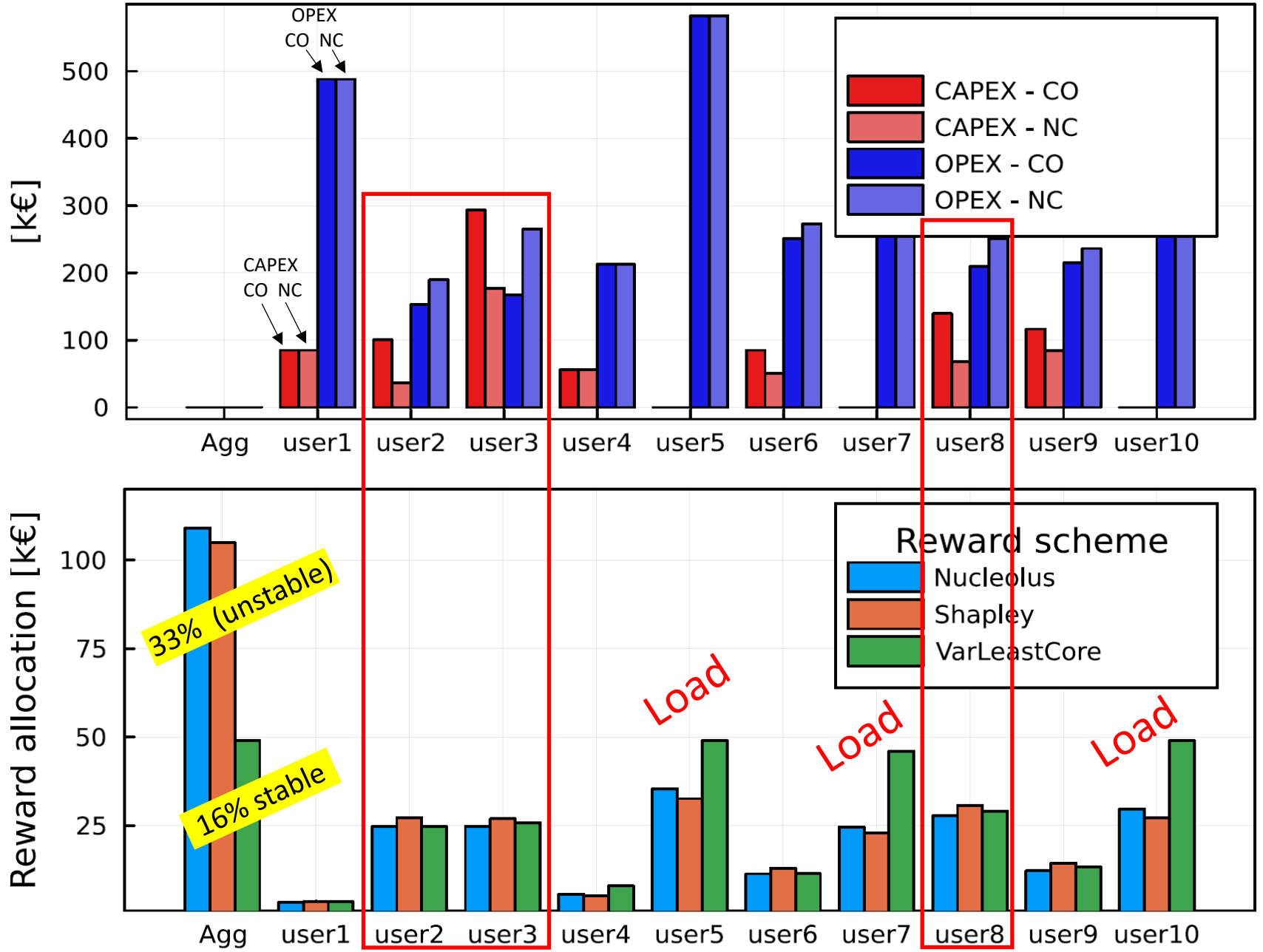
EnergyCommunity.jl

julia

**Cooperative game theory**

TheoryOfGames.jl

julia




33% aggr.
30% loads
37% gen.
(fair, unstable)

16% aggr.
48% loads
36% gen.
(stable; fair??)

# Some simplified approaches

## Energy Sharing (SH) scheme

Reward proportional to
shared consumption and production

$$R_j^{SH} = \pi^{SH+} E_j^{SH+} + \pi^{SH-} E_j^{SH-}$$

Shared production

Shared consumption

## Financial (F) scheme

Reward proportional
to the investment

$$R_j^F = \gamma^F CAPEX_j$$

## Energy scheme

Fixed quote + quote proportional
to consumption or production

$$R_j^E = \gamma^0 + \gamma^L \frac{\sum_t E_{j,t}^L}{\sum_{j,t} E_{j,t}^L} + \gamma^P \frac{\sum_t E_{j,t}^P}{\sum_{j,t} E_{j,t}^P}$$

## Asset-based scheme

Reward proportional
to the size of plants

$$R_j^A = \sum_a \gamma_a^A Dx_j^{a,U}$$

**How to find easy and transparent tuning rules to align them to the results of Game Theory?**

# Aligning simplified allocation rules to Variance Least Core

| Ref. reward | Case | RMSE k€ | LeastCore $\vartheta$ k€ | $\gamma^0$ %* | $\gamma^L$ %* | $\pi_{SH+}$ €/kWh | $\pi_{SH-}$ €/kWh | $\gamma^F$ %/k€* | $\gamma^A_{PV}$ %/kWp* | $\gamma^A_{Batt}$ %/kWh* | $\gamma^A_{Wind}$ %/kWp* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| VarLeastCore | Base | 15.61 | -8.66 | 8.41 | - | - | - | - | - | - | - |
| | Demand | 17.4 | -13.14 | - | 83.47 | - | - | - | - | - | - |
| | EnergySh. | 6.12 | 12.73 | 0.0 | - | 0.03 | 0.06 | - | - | - | - |
| | Install (1) | 15.61 | -8.66 | 8.41 | - | - | - | - | 0.0 | 0,0 | 0.0 |
| | Install (2) | 15.61 | -8.66 | 8.41 | - | - | - | 0.0 | - | - | - |
| | Mixed (1) | 4.99 | 11.82 | 0.0 | - | 0.02 | 0.06 | 0.05 | - | - | - |
| | Mixed (2) | 4.58 | 10.9 | 0.0 | - | - | 0.06 | - | 0.15 | 0.34 | 0.19 |
| | Complete | 4.58 | 10.92 | 0.0 | 0.0 | 0.0 | 0.06 | 0.0 | 0.15 | 0.34 | 0.19 |

*: percentage value with respect to the total annualized reward (308 k€).

- Some simplified allocation schemes are unstable
- Other criteria pose fairness concerns
- The “complete” scheme is promising
  (to be improved using a more balanced -and fairer- VarLeastCore reference scheme)

*Work in progress…*

# Conclusions


- A tool to optimize ECs and quantify their benefits
- **Game Theory** is highly complex and difficult to implement, as it is, for reward distribution
- **Simplified reward schemes** are promising, but need proper tailoring

## Next steps

- Tuning the reference VLC allocation to increase its fairness
- Stoch. optimization of the participation of ECs to energy markets
- Deploying the contribution of ECs to local flexibility markets

# Thank you!

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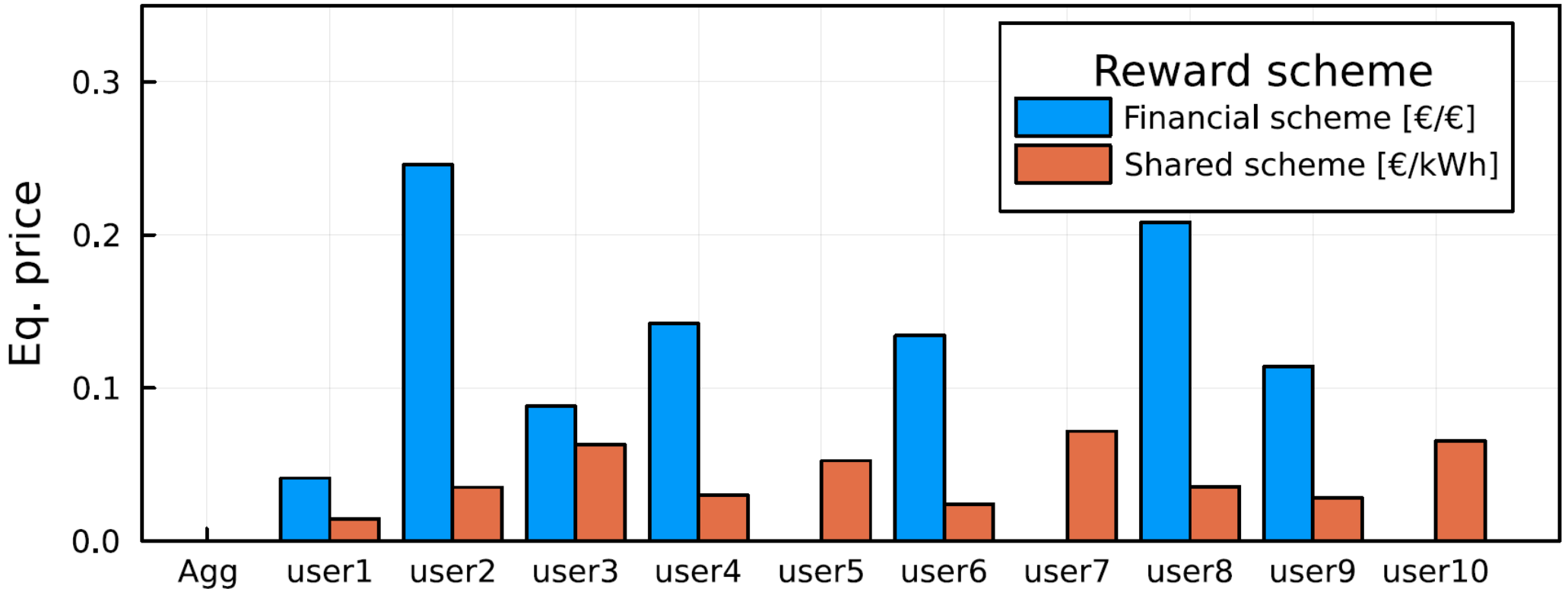
# Aligning Shared Energy and Financial schemes with VarLeastCore…




- The Financial scheme equivalent to VLC requires CAPEX discounts ranging from 4 to 25%
- The Shared Energy scheme equivalent to VLC requires rewards ranging from 1 to 7 c€/kWh$_{shared}$

D.Fioriti , T.Ferrucci, D.Poli, «Fairness and reward in Energy Communities: game-theory versus simplified approaches, *EEEIC 2023*, Madrid, June 2023.