

Optimal investment in storage in a distributed renewable energy system.

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- Power generation from Renewable Energy Sources (RES) is crucial for **the energy transition**
- Distributed generation of RES power **for self consumption** is increasing
- However, RES are **non-controllable** → need of storage

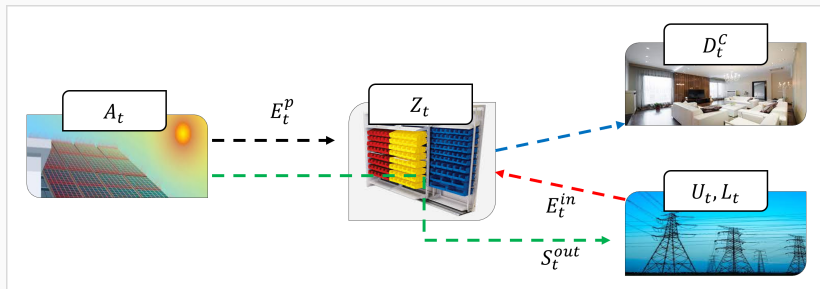
Perspective

Decision making process of an **energy manager** (owner / external manager), who needs to **set up a distributed energy system**

Aim

What is the **optimal investment** in a Battery Energy **Storage System**, BESS, (size/capacity) with power production from **RES** in a **dynamic pricing** environment?

The storage system and the flow of energy



1. A_t : cumulative distribution of renewable energy (PV) produced up to t (random);
2. E_t^p : average output per unit of time t (then hp: constant $E_t^p = E^p$);
3. $Z_t \in [\underline{Z}; \bar{Z}]$: electricity stored at time t ;
4. D_t^c : average demand at time t (then constant by hp: $D_t^c = D^c$);
5. $E_t^{in}(L_t)$: energy purchased from the grid at time t , depending on L_t , i.e. the energy imported from the grid to maintain storage at the minimum level \underline{Z} ;
6. $S_t^{out} = (U_t)$: energy sold to the grid at time t , depending on U_t , i.e. the amount of energy in excess of the maximum storage capacity \bar{Z} fed into the grid up to time t .

In brief, storage implies an opportunity cost.

- Power can be purchased (at a certain price), generated or sold (at a certain price).
- If stored, power cannot be sold.
- If purchased, power needs to be paid.
- Load needs to be served.

Assumptions.

1. Energy production A_t and consumption B_t ($D_t^c = D^c$) are random.
2. The system gives priority to domestic consumption D^c (proximity).
3. Storage cannot go below \underline{Z} and above \bar{Z} .
4. Prices are random: the purchasing price, P_t^b , is higher than the selling price P_t^s (e.g., system costs).

The *amount of energy stored in the battery* system at time t is:

$$Z_t = Z_{t-1} + \underbrace{(A_t - A_{t-1})}_{E^P} - \underbrace{(B_t - B_{t-1})}_{D^C} + \underbrace{\max[L_t - L_{t-1}, 0]}_{E_t^{in}} - \underbrace{\max[U_t - U_{t-1}, 0]}_{S_t^{OUT}} \quad (1)$$

→ A_t , the cumulative distribution of *energy produced* up to time t , evolves according to an ABM:

$$A_t = A_0 + E^P t + \sigma^A W_t^A \quad W_t^A \sim N(0, t) \quad (2)$$

→ B_t , the cumulative of *energy demanded* up to time t , evolves according to an ABM:

$$B_t = B_0 + D^C t + \sigma^B W_t^B \quad W_t^B \sim N(0, t) \quad (3)$$

with $\text{cov}(W_t^A, W_t^B) = 0$

The stochastic storage system

Assuming no correlation between energy production (2) and energy demand (3), the *stochastic process describing the amount of energy stored* at t is:

$$Z_t = Z_0 + (nS)t + \sigma W_t^Z + L_t - U_t \quad (4)$$

where:

- $Z_0 = A_0 - B_0 \geq \underline{Z}$
- $W_t^Z = W_t^A - W_t^B \sim N(0, t)$,
- $\sigma = \sqrt{(\sigma^A)^2 + (\sigma^B)^2}$
- $nS = E^P - D^C$ indicates the average *net-Supply*

Finally, in a **continuous-time framework**, we can write (4) as follows:

$$dZ_t = nSdt + \sigma dW_t^Z + dL_t - dU_t \quad Z(0) = Z_0 \quad (5)$$

Energy has a value that depends on:

- i) the cost of buying energy from the national grid,
- ii) the value of self-generated power, which depends on:
 - iii) the opportunity cost of using it for satisfying the load or keeping it stored into the battery
 - iv) the revenues obtained by feeding it into the national grid

Energy buying price. Whenever $Z_t < \underline{Z}$, energy must be purchased at price P_t^b driven by a GBM:

$$dP_t^b = \gamma P_t^b dt + \sigma_t^b P_t^b dW_t^b \text{ with } P^b(0) = P_0^b \quad (6)$$

Energy selling price. Whenever $Z \geq \bar{Z}$, energy is sold to the grid at price P_t^s , which we model as a function of the buying price P_t^b

$$P_t^s = (1 + k)P_t^b. \quad (7)$$

$k \in [-1, 0]$ captures relation among prices, such that $P_t^s < P_t^b$.

The value of the energy stored in the battery, is a function of P_t^b :

$$P_t^z = \alpha P_t^b + (1 - \alpha)P_t^s = (1 + (1 - \alpha)k)P_t^b \quad \alpha \in [0, 1] \quad (8)$$

→ an increase in k , i.e. an increase in the value of energy sold to the grid, implies an increase in the value of energy stored into the battery.

The optimization problem

The problem consists in identifying the *optimal BESS capacity level* such that $F(\underline{Z})$ is minimized, i.e. the expected value of the sum of the **opportunity cost of keeping the energy into the BESS** plus the **power purchase costs** net of **revenues obtained from selling excess energy to the grid** over a planning period that we approximate to infinity

$$\begin{aligned} F(\underline{Z}) &= E \left[\int_0^\infty e^{-\rho t} P_t^z Z_t + \int_0^\infty e^{-\rho t} P_t^b dL_t - \int_0^\infty e^{-\rho t} P_t^s dU_t \right] \quad (9) \\ &= P_0^b E \left[\int_0^\infty e^{-rt} [(1 + (1 - \alpha)k)Z_t + dL_t - (1 + k)dU_t] \right] \end{aligned}$$

subject to the dynamics of the energy stored in the battery:

$$dZ_t = nSdt + \sigma dW_t^z + dL_t - dU_t \quad (10)$$

We determine the *optimal storage capacity level* \bar{Z}^* such that

- the process Z_t *maintains within the boundaries* $[\underline{Z}, \bar{Z}]$
- the minimum effort is exerted, i.e. *we satisfy end-user load and reduce the need of buying energy* from the grid.

The optimization problem

Given the initial level of energy in the battery $Z_0 \geq \underline{Z}$, the *minimum cost of operating the BESS* is:

$$F(Z_0) = \frac{P_0^b(1 + (1 - \alpha)k)}{r} \left(Z_0 + \frac{nS}{r} \right) + P_0^b \left[\frac{v(e^{\eta_2(\bar{Z}^* - \underline{Z})} - 1) - k}{\eta_1(e^{\eta_1(\bar{Z}^* - \underline{Z})} - e^{\eta_2(\bar{Z}^* - \underline{Z})})} e^{\eta_1(Z_0 - \underline{Z})} + \frac{k + v(1 - e^{\eta_1(\bar{Z}^* - \underline{Z})})}{\eta_2(e^{\eta_1(\bar{Z}^* - \underline{Z})} - e^{\eta_2(\bar{Z}^* - \underline{Z})})} e^{\eta_2(Z_0 - \underline{Z})} \right] \quad (11)$$

where $v = \frac{r + (1 + (1 - \alpha)k)}{r} > 0$, $\eta_1 = \frac{-nS + \sqrt{(nS)^2 + 2\sigma^2 r}}{\sigma} > 0$ and $\eta_2 = \frac{-nS - \sqrt{(nS)^2 + 2\sigma^2 r}}{\sigma^2} < 0$ and:

- is the BESS opportunity cost if there were no controls given by its technical limits
- is the operating cost, i.e. cost of buying energy from the grid when Z_t approaches \underline{Z} , net of the expected revenues obtained by selling energy to the grid when Z_t crosses \bar{Z}^*

The optimization problem

The *optimal battery capacity* \bar{Z}^* is given by the solution of the following:

$$v(\eta_1 - \eta_2) e^{(\eta_2 + \eta_1)(\bar{Z}^* - \underline{Z})} - (v + k) \left[\eta_1 e^{\eta_1(\bar{Z}^* - \underline{Z})} - \eta_2 e^{\eta_2(\bar{Z}^* - \underline{Z})} \right] = 0 \quad (12)$$

where $\bar{Z}^* > \underline{Z}$.

The *optimal operating cost* is:

$$F(\underline{Z}) = \frac{P_0^b(1 + (1 - \alpha)k)(nS)}{r^2} + \frac{P_0^b(1 + (1 - \alpha)k)\underline{Z}}{r} \quad (13)$$
$$+ \frac{P_0^b}{(e^{\eta_1(\bar{Z}^* - \underline{Z})} - e^{\eta_2(\bar{Z}^* - \underline{Z})})} \frac{\sqrt{(nS)^2 + 2\sigma^2 r}}{((1 + r)(1 + k) - \alpha k)} \left[(v)^2 e^{\frac{-2nS}{\sigma^2}(\bar{Z}^* - \underline{Z})} - (v + k)^2 \right]$$

...need for a numerical solution...

Before numbers... what can we say via comparative statics on nS

An average production (E^P) above the average demand D^C (i.e. $nS = E^P - D^C > 0$) favors the selling of energy to the grid, consequently, this generates a **reduction** in the expected operating cost $F(\underline{Z})$.

Conversely, an average production lower than the average demand (i.e. $nS < 0$) increases the amount energy purchased from the grid, this generates an **increase** in the expected BESS operating cost $F(\underline{Z})$.

When production matches perfectly demand, i.e. $nS = 0$, the optimal maximum capacity yields from :

$$e^{\sqrt{\frac{2r}{\sigma^2}}(\bar{z}^* - \underline{z})} - e^{-\sqrt{\frac{2r}{\sigma^2}}(\bar{z}^* - \underline{z})} - 2\frac{v}{v+k} = 0 \quad (14)$$

and the minimum BESS operating cost is:

$$F(\underline{Z}) = \frac{P_0^b(1 + (1 - \alpha)k)\underline{Z}}{r} + P_0^b\sqrt{\frac{2\sigma^2}{r}} \left[\frac{(-k)(2v + k)}{2v} \right] \quad (15)$$

Before numbers... what can we say via comparative statics on k

Parameter $k \in [-1, 0]$ captures that the selling price is, in general, less than the buying price. Its effect on the optimal capacity of the battery \bar{Z}^* is:

$$\frac{\partial \bar{Z}^*}{\partial k} < 0 \quad (16)$$

If the selling price of energy P_t^s increases (i.e., k increases):

- the optimal BESS capacity reduces ($\downarrow \bar{Z}^*$);
- the optimal strategy is to **sell energy** to the grid while storing as **little energy as possible**, i.e. \underline{Z} ;
- the cost of **buying** energy from the grid in the future is more than **offset by the benefit of selling** excess energy to the grid.

Conversely, *if the selling price decreases, (i.e., k decreases):*

- there is **no advantage in selling** energy to the grid, but this **increases the value of energy stored**.
- This, in turn, induces the REC manager to **increase the BESS capacity**.

Let me recall that $\sigma = \sqrt{(\sigma^A)^2 + (\sigma^B)^2}$ and it drives the dynamics of storage levels at each time t

$$Z_t = \underline{Z} + nSt + \sigma W_t^Z + L_t - U_t \quad (17)$$

$$dZ_t = nSdt + \sigma dW_t^Z + dL_t - dU_t \quad (18)$$

Consequently:

$$\frac{\partial \bar{Z}^*}{\partial \sigma} > 0 \quad (19)$$

→ The REC manager reduces operating costs by increasing the BESS capacity (hedge against the risk of buying energy at a buying price higher than its selling price .

Based on parameters in Table 1, we **determine the optimal maximum level of energy to be stored in the battery \bar{Z}^*** .

First, we identify the difference between \bar{Z}^* and \underline{Z} as the solution of eq. (12), and (14) if $nS = 0$, under different cases.

The results can also be related to \bar{Z}^* by assuming $\underline{Z} = 0$.

<i>Parameter</i>	<i>Value</i>	<i>Description</i>
σ_A	0.15	volatility of the cumulative distribution of energy produced; assumption
σ_B	0.05	volatility of the cumulative distribution of energy needed by the user; assumption
σ	0.20	computation as $\sigma = \sigma_A + \sigma_B$
k	-0.10	relation among buying and selling prices
r	0.10	risk-adjusted discount rate; assumption
nS	0.01	net energy supply; assumption of excess of production
P_0^b	59.21	initial value of the buying price; [Bonaldo et al., 2024]

Table 1: Calibration

We investigate several contexts:

- $\alpha = \{0, 0.5, 1\}$ → the energy buying P_t^b and selling prices P_t^s have different weights in determining P_t^z , i.e. the value of energy stored in the battery;
- $k = \{-0.1; -0.3; -0.5\}$, an increase in k implies an increase in the value of energy sold to the grid and thus a higher value of energy stored;
- $1 + (1 - \alpha)k = 0$: energy can be stored for free;
- $1 + (1 - \alpha)k = \{0.3; 0.5; 0.9\}$: there is an opportunity cost of maintaining energy stored;

to study following cases:

- A. a decrease in the risk-adjusted discount rate r from 0.10 to 0.05;
- B. case A & an increase in volatility σ from 0.20 to 0.40;
- C. a decrease in the risk-adjusted discount rate to 0.01, an increase in σ to 0.40 and nil net energy supply ($nS = 0$)
- D. case C & $k = -0.50$

Table 2: Results for $\bar{z}^* - \underline{z}$ with $\alpha = \{0; 0.5; 1\}$ (highest/lowest levels in red).

α	Table 1	Case A	Case B	Case C	Case D
		$r = 0.05$	$r = 0.05,$ $\sigma = 0.40$	$r = 0.01,$ $\sigma = 0.40,$ $nS = 0$	$r = 0.01,$ $\sigma = 0.40,$ $nS = 0,$ $k = -0.5$
0	0.0631	0.0647	0.1297	0.1327	0.3977
0.5	0.0617	0.0630	0.1264	0.1292	0.3254
1	0.0602	0.0615	0.1234	0.1291	0.2820

- As α increases, the optimal quantity of energy stored reduces.
- The highest level of energy stored in the battery is reached when $\alpha = 0$ in Case D.

Numerical part - Preliminary results: $F(\underline{Z})$

Table 3: Results for optimal cost $F(\underline{Z})$ with $\underline{Z} = \{0; 0.10\}$ $\alpha = \{0; 0.5; 1\}$

$F(\underline{Z}, \alpha)$	Table 1	Case A $r = 0.05$	Case B $r = 0.05,$ $\sigma = 0.40$	Case C $r = 0.01,$ $\sigma = 0.40,$ $nS = 0$	Case D $r = 0.01,$ $\sigma = 0.40,$ $nS = 0,$ $k = -0.5$
$F(0.10, 0)$	85.1350	168.4545	241.3292	566.3658	462.7002
$F(0, 0)$	31.8460	61.8765	134.7512	33.4758	166.6502
$F(0.10, 0.5)$	88.9887	176.2800	251.0509	595.9718	610.9953
$F(0, 0.5)$	32.7392	63.7814	138.5519	33.47679	166.9203
$F(0.10, 1)$	92.8573	184.0603	260.6757	625.5777	759.1566
$F(0, 1)$	33.6473	65.6403	142.2557	33.4776	167.0566

- As α increases, cost $F(\underline{Z})$ increases.
- The highest cost $F(\underline{Z})$ is reached when when $\alpha = 0$ in Case D.
- The shift from a positive to a nil minimum level of energy stored affect significantly the optimal cost
- The maximum cost is reached in Case D when $\alpha = 1$.

Numerical part - Preliminary results: $1 + (1 - \alpha)k$ with $r = 0.05$

We now study $\bar{Z}^* - \underline{Z}$ by considering the relationship between the value of energy stored P_t^z and the buying price of energy P_t^b (i.e., $1 + (1 - \alpha)k$) in different scenarios and for different k .

$1 + (1 - \alpha)k$	Case A $k = -0.1$	Case B $k = -0.3$	Case C $k = -0.5$	Case D $k = -0.1,$ $nS = 0$
0.9	0.0646	0.1121	0.1449	0.0650
0.5	0.0856	0.1494	0.1944	0.0856
0.3	0.1075	0.1886	0.2465	0.1075
0	0.2954	0.5664	0.8329	0.2954

Table 4: Results for $\bar{Z}^* - \underline{Z}$ with $r = 0.05$, $\sigma = 0.20$ and $nS = 0.01$

- As the cost of storing energy reduces i.e., $1 + (1 - \alpha)k \rightarrow 0$, the optimal quantity of energy stored increases;
- The less is k , the greater this effect;
- The absence of excess production does not have a significant impact on the optimal quantity of energy to be stored.

Numerical part - Preliminary results: $1 + (1 - \alpha)k$ with $r = 0.05$

$F(\underline{Z}; 1 + (1 - \alpha)k)$	Case A $k = -0.1$	Case B $k = -0.3$	Case C $k = -0.5$	Case D $k = -0.1,$ $nS = 0$
$F(0.10; 0.9)$	168.4546	223.0159	260.8674	114.0478
$F(0; 0.9)$	61.8765	116.4379	154.2894	7.4698
$F(0.10; 0.5)$	103.8542	144.9201	172.3318	66.6655
$F(0; 0.5)$	44.64416	85.7101	113.1218	7.4555
$F(0.10; 0.3)$	96.22362	131.9376	156.7697	42.9620
$F(0; 0.3)$	60.69762	96.4116	121.2437	7.4360
$F(0.10; 0)$	30.37223	44.45676	55.44826	7.1151
$F(0; 0)$	30.37223	44.45676	55.44826	7.1151

Table 5: Results for $F(\underline{Z})$ with $\underline{Z} = \{0; 0.10\}$, $r = 0.05$, $\sigma = 0.20$ and $nS = 0.01$

- The less is k , the greater the optimal cost.
- When the minimum battery level is zero and there is no cost (or gain) in storing energy, the least cost is achieved, though this is not the case where it is optimal to maintain the minimum energy stored.

Numerical part - Preliminary results: $1 + (1 - \alpha)k$ and changes in r

As the risk-adjusted discount rate r decreases, the optimal quantity of energy stored increases.

$1 + (1 - \alpha)k$	Case A $k = -0.1$	Case B $k = -0.3$	Case C $k = -0.5$	Case D $k = -0.1,$ $nS = 0$
0.9	0.0646	0.1121	0.1449	0.0650
0.5	0.0856	0.1494	0.1944	0.0856
0.3	0.1075	0.1886	0.2465	0.1075
0	0.2954	0.5664	0.8329	0.2954

Table 6: Results for $\bar{Z}^* - \underline{Z}$ with $r = 0.05$, $\sigma = 0.20$ and $nS = 0.01$

$1 + (1 - \alpha)k$	Case A $k = -0.1$	Case B $k = -0.3$	Case C $-k = 0.5$	Case D $k = -0.1,$ $nS = 0$
0.9	0.0653	0.1130	0.1458	0.0657
0.5	0.0871	0.1515	0.1966	0.0871
0.3	0.1105	0.1929	0.2510	0.1105
0	0.3814	0.7313	1.0753	0.3814

Table 7: Results for $\bar{Z}^* - \underline{Z}$ with $r = 0.03$, $\sigma = 0.20$ and $nS = 0.01$

Numerical part - Preliminary results: $1 + (1 - \alpha)k$ and changes in r

As the risk-adjusted discount rate r decreases, the optimal cost $F(\underline{Z})$ increases.

Table 8: Results for $F(\underline{Z})$ with $\underline{Z} = \{0; 0.10\}$, $r = \{0.05; 0.03\}$, $\sigma = 0.20$ and $nS = 0.01$ (selected cases)

$F(\underline{Z}; 1 + (1 - \alpha)k)$	Case A $k = -0.1$	Case B $k = -0.3$	Case C $k = -0.5$	Case D $k = -0.1,$ $nS = 0$
$F(0.10; 0.9)$ with $r = 0.05$	168.4546	223.0159	260.8674	114.0478
$F(0; 0.9)$ with $r = 0.05$	61.8765	116.4379	154.2894	74698
$F(0.10; 0.5)$ with $r = 0.05$	103.8542	144.9201	172.3318	66.6655
$F(0; 0.5)$ with $r = 0.05$	44.64416	85.7101	113.1218	74555
$F(0.10; 0.9)$ with $r = 0.03$	279.5264	369.8511	432.6078	187.2834
$F(0; 0.9)$ with $r = 0.03$	101.8964	192.2211	254.9778	9.6533
$F(0.10; 0.5)$ with $r = 0.03$	170.6946	240.9112	288.0857	108.3249
$F(0; 0.5)$ with $r = 0.03$	72.01129	142.2279	189.4023	9.641588

Numerical part - Preliminary results: increasing volatility

As the volatility of energy stored increases, the optimal quantity of stored energy increases too.

$1 + (1 - \alpha)k$	Case A $k = -0.1$	Case B $k = -0.3$	Case C $-k = 0.5$	Case D $k = -0.1,$ $nS = 0$
0.9	0.0653	0.1130	0.1458	0.0657
0.5	0.0871	0.1515	0.1966	0.0871
0.3	0.1105	0.1929	0.2510	0.1105
0	0.3814	0.7313	1.0753	0.3814

Table 9: Results for $\bar{Z}^* - \underline{Z}$ with $r = 0.03$, $\sigma = 0.20$ and $nS = 0.01$

$1 + (1 - \alpha)k$	Case A $k = -0.1$	Case B $k = -0.3$	Case C $k = -0.5$	Case D $k = -0.1,$ $nS = 0$
0.9	0.1310	0.2270	0.2935	0.1313
0.5	0.1742	0.3031	0.3932	0.1742
0.3	0.2210	0.3858	0.5020	0.2210
0	0.7628	1.4625	2.1506	0.7628

Table 10: Results for $\bar{Z}^* - \underline{Z}$ with $r = 0.03$, $\sigma = 0.40$ and $nS = 0.01$

Numerical part - Preliminary results: increasing volatility

As the volatility of energy stored increases, the optimal quantity of stored energy increases too, *along with with the optimal cost.*

Table 11: Results for $F(\underline{Z})$ with $\underline{Z} = \{0; 0.10\}$, $r = 0.03$, $\sigma = \{0.20; 0.40\}$ and $nS = 0.01$ (selected cases)

$F(\underline{Z}; 1 + (1 - \alpha)k)$	Case A $k = -0.1$	Case B $k = -0.3$	Case C $k = -0.5$	Case D $k = -0.1,$ $nS = 0$
$F(0.10; 0.9)$ with $\sigma = 0.20$	279.5264	369.8511	432.6078	187.2834
$F(0; 0.9)$ with $\sigma = 0.20$	101.8964	192.2211	254.9778	9.6533
$F(0.10; 0.5)$ with $\sigma = 0.20$	170.6946	240.9112	288.0857	108.3249
$F(0; 0.5)$ with $\sigma = 0.20$	72.01129	142.2279	189.4023	9.641588
$F(0.10; 0.9)$ with $\sigma = 0.40$	399.7532	577.837	700.6201	196.9367
$F(0; 0.9)$ with $\sigma = 0.40$	222.1232	400.207	522.9901	19.3067
$F(0.10; 0.5)$ with $\sigma = 0.40$	262.0527	394.8389	484.6461	117.9665
$F(0; 0.5)$ with $\sigma = 0.40$	163.3694	296.1555	385.9628	19.28318

Summary of results.

- We provide a theoretical model to assess the optimal maximum level of a battery energy storage system in a dynamic pricing environment.
- Comparative statistics combined with a numerical example show the relevance of the relationship between the buying and selling price of energy at the maximum level of the battery, and shed light on the effect of volatility on BESS size.

Work to be done.

- Comprehensive discussion of the results and policy implications (i.e. energy savings, subsidies, capacity market).

Thank you for your attention!

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