

Bidding and Investment in Wholesale Electricity Markets

Pay-as-Bid versus Uniform-Price Auctions

Bert Willems^{1,2} Yueting Yu¹

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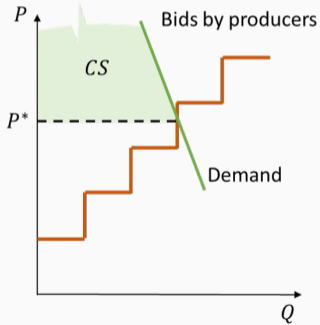
¹Tilburg University

²Toulouse School of Economics

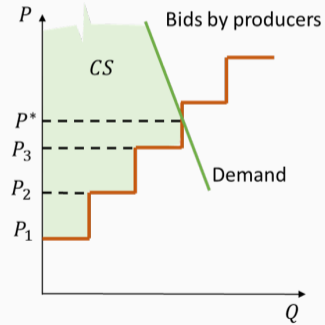
Wholesale Electricity Markets

- Electricity is a non-storable commodity → supply must meet demand at all times.
- Two ways to organize wholesale market:

(a) A uniform-price auction (UPA).



(b) A pay-as-bid auction (PABA).



- Which method is better?

- We compare those two multi-unit auction formats.
- In the **short term**: bidding behaviors and price-cost mark-ups.
- In the **long term**: investment and generation portfolio. ← novel
- Construction of perfect competition model with
 - uncertain and elastic demand,
 - a continuum of generation technologies (from base-load to peak-load).

Literature: Auction Theory on PABA

- We are not the first to compare PABA and UPA:

	Model	Investment	CS	Welfare
Federico & Rahman '03	perf.comp/monop.	no	+	-
Holmberg '09	SFE	no	+	=
Fabra et al. '06	duopoly-step	no	+	=
Fabra et al. '11	duopoly-step	1 tech	+	=
Our paper	perf. comp.	∞ tech	-	-

- **Short-term:** in equilibrium firms submit bids $> MC$. Hence, $WTP > MC$.
→ distorts consumption decision
- **Long-term:** revenue of base-load producers is depressed during high demand
→ distorts generation mix.

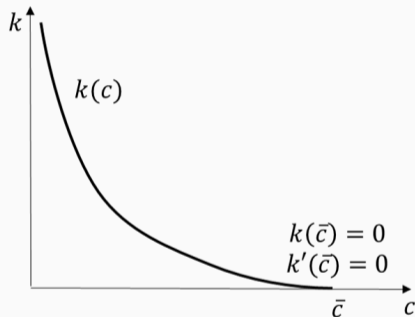
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Model

Model Set-up: Supply

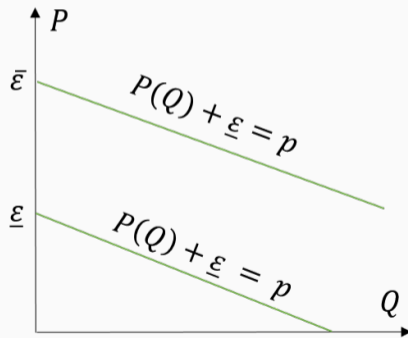
- Continuous set of technologies with marginal cost $c \in (0, \bar{c}]$ with \bar{c} the VOLL.
- Technology frontier: convex capital cost function $k(c)$



- Infinitely many small firms can invest in technology c
- Total expected profit: $\pi(c) = T(b(c)) - k(c) - c \cdot H(b(c))$:
 - expected transfers: $T(b(c))$,
 - expected capacity factor: $H(b(c))$.

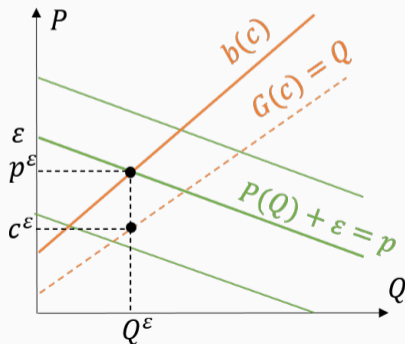
Model Set-up: Demand

- Consumers are price takers.
- Stochastic and elastic inverse demand function: $p = P(q) + \varepsilon$.
Normalized such that ε is the intercept of the demand function ($P(0) = 0$).
- Demand shock ε distributed with CDF $F(\varepsilon)$ over $[\underline{\varepsilon}, \bar{\varepsilon}]$.
Quantile function $Q(\cdot) = F^{-1}(\cdot)$.



Model Set-up: Market Clearing

- Bidding and investment **strategies** $\{b(c), G(c)\}$:
 - $b(c)$ bids by firm with marginal cost c . Assume $b'(c) > 0$.
 - $G(c)$ total installed capacity with marginal costs equal or less than c .
- **Market clearing** then determines clearing price p^ε , quantity Q^ε , and marginal power plant c^ε for any given demand shock ε .



Model Set-up: Market Clearing

- We will index the different states of the world not by the demand shock ε but by the marginal power plant c (firm's type).
- The **market clearing condition** when firm of type c is marginal is

$$p(c) = b(c) = P(G(c)) + \varepsilon(c),$$

This determines $\varepsilon(c)$, the demand shock for which firm of type c is marginal.

- The **capacity factor** $h(c)$ of a firm of type c is then given by

$$h(c) = 1 - F(\varepsilon(c)).$$

- The **expected revenue** $T(c)$ of a firm of type c under uniform price and pay-as-bid auctions:

$$T^{\text{up}}(c) = \int_c^{\bar{c}} b(t) dh(t), \quad T^{\text{pab}}(c) = b(c)h(c).$$

Competitive Market Equilibrium

- What constitutes a competitive bidding and investment equilibrium $\{b(c), G(c)\}$?
- Assumptions:
 - Producers invest and bid before the demand shock is realized (long-lasting bids).
 - Producers are price-takers: they take the stochastic distribution of prices as given.
 - No entry barriers.
- Competitive Market Equilibrium:
 - **Short-run:** firm sets $b(c)$ to maximize profit for a given stochastic price distribution with CDF $Z(p)$. This price distribution is consistent with market clearing:

$$Z(p(c)) = F(\varepsilon(c)).$$

- **Long-run:** firm makes zero expected profit $\pi(c) = 0$.

Analysis

I. Bidding Equilibrium

- The optimal bidding strategies follow the FOC (Federico & Rahman, 2003):

$$b^{\text{UP}}(c) = c, \quad b^{\text{PAB}}(c) = c + \frac{1 - Z(b^{\text{PAB}}(c))}{Z'(b^{\text{PAB}}(c))}.$$

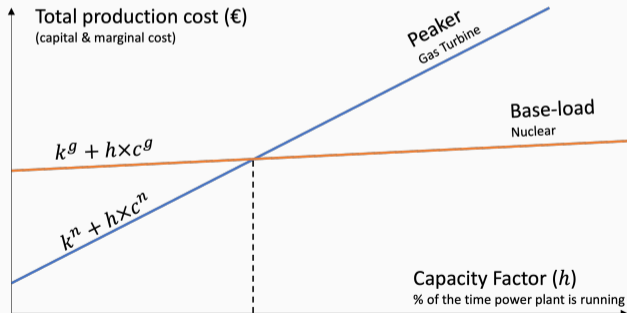
- PAB: trade-off between mark-up and being scheduled (similar to 1st price auction).
- However, the price distribution $Z(p)$ is endogenous and depends on $b(c)$.
- Hence, the optimal bid $b(c)$ and the capacity factor $h(c)$ are determined by a differential equation.

II. Investment Equilibrium

- Independent of auction format, from the envelope theorem the capacity factor $h(c)$ satisfies

$$h(c) = -k'(c).$$

- Intuition:** Screening curves - which technology is the cheapest depends on capacity factor h (Stoft, 2002; Boiteux, 1949).



$$h = -\frac{\Delta k}{\Delta c} = -\frac{k^g - k^n}{c^g - c^n}$$

II. Investment Equilibrium (cont'd)

- Firm with technology c bids its marginal cost (in UPA) or levelized cost (in PABA)

$$b^{\text{UP}}(c) = c, \quad b^{\text{PAB}}(c) = c + \frac{k(c)}{h(c)}.$$

- The Lerner index PABA is the reciprocal of the elasticity $\epsilon_k(c)$ of investment costs:

$$L = \frac{b(c) - c}{c} = \frac{k(c)}{|k'(c)|c} := \frac{1}{\epsilon_k(c)}.$$

Not due to market power, but necessary to recoup investment costs.

- The cumulative installed capacity $G(c)$ can be calculated from market clearing condition.

Example

A Functional-Form Model: Assumptions

- Linear demand function

$$P(q) = -\rho q \quad \text{with } \rho > 0.$$

- Convex investment cost

$$k(c) = \frac{1}{\gamma + 1} \frac{(\bar{c} - c)^{\gamma+1}}{\bar{c} - \underline{c}} \quad \text{with } \gamma \in (0, 1).$$

- Uniformly distributed demand shocks over $[\underline{\varepsilon}, \bar{\varepsilon}]$.

Producers' optimal bidding strategy and investment decision

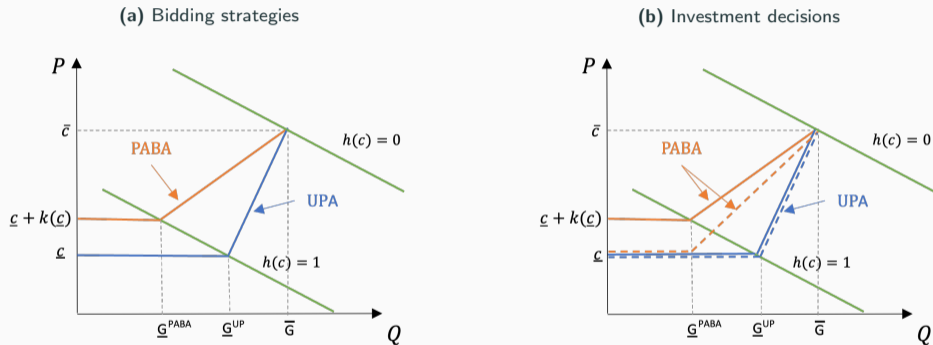


Figure 2: Comparison of bidding (solid) and portfolios (dashed) between UPA and PABA.

Comparison of PABA versus UPA

- Investments

- Aggregate investments are identical in the two auctions as in Fabra et al. (2011), $\bar{G}^{UP} = \bar{G}^{PAB}$.
- But the generation mix is distorted.
- Fewer investments in the baseload capacity ($\underline{G}^{PAB} < \underline{G}^{UP}$).
- More investments in all intermediate technologies ($G'^{PAB} > G'^{UP}$).

- All firms make zero profit (free entry), so welfare = CS.

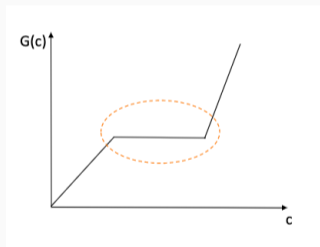
- The UPA is efficient (= Peak-load pricing, Boiteux (1949)), so $CS^{UP} > CS^{PAB}$.
- CS with high demand is higher: as volume is the same & consumers pay less.
- CS with low demand is lower: as volume is smaller & price is higher.

Summary and Future Research

Summary of Results

- Our research speaks to the question how auction formats affect short-term (bidding) and long-term (investment incentives) decisions.
- Inefficiency does not necessarily originate from market power. It could come from market design. Under PABA,
 - In the short run, consumers' WTP is higher than producers' marginal costs.
= **Allocative inefficiency**
 - In the long run, revenue for baseload is distorted downwards, and incentives for investment decrease.
= **Distortion in generation mix**

- Allow for some bunching, i.e. $G'(c) = 0$ for some c ? Some intermediate technologies are not used.



- Introduce market power in a monopoly setting.

Thank you :)