Bidding and Investment in Wholesale Electricity Markets

Pay-as-Bid versus Uniform-Price Auctions

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16 December 2021 - AIEE Symposium

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Wholesale Electricity Markets

- Electricity is a non-storable commodity → supply must meet demand at all times.
- Two ways to organize wholesale market:

  (a) A uniform-price auction (UPA).

  (b) A pay-as-bid auction (PABA).

- Which method is better?
This Paper

• We compare those two multi-unit auction formats.
• In the **short term**: bidding behaviors and price-cost mark-ups.
• In the **long term**: investment and generation portfolio. ← novel
• Construction of perfect competition model with
  - uncertain and elastic demand,
  - a continuum of generation technologies (from base-load to peak-load).
We are not the first to compare PABA and UPA:

<table>
<thead>
<tr>
<th>Model</th>
<th>Investment</th>
<th>CS</th>
<th>Welfare</th>
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<tbody>
<tr>
<td>Federico &amp; Rahman ’03</td>
<td>perf.comp/monop.</td>
<td>no</td>
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<td>Holmberg ’09</td>
<td>SFE</td>
<td>no</td>
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<td>Fabra et al. ’06</td>
<td>duopoly-step</td>
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<td>Fabra et al. ’11</td>
<td>duopoly-step</td>
<td>1 tech</td>
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<tr>
<td>Our paper</td>
<td>perf. comp.</td>
<td>∞ tech</td>
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- **Short-term**: in equilibrium firms submit bids $> MC$. Hence, WTP $> MC$.
  $\rightarrow$ distorts consumption decision

- **Long-term**: revenue of base-load producers is depressed during high demand
  $\rightarrow$ distorts generation mix.
1. Model

2. Analysis
   I. Bidding Equilibrium
   II. Investment Equilibrium

3. Example

4. Summary and Future Research
Model
Model Set-up: Supply

- Continuous set of technologies with marginal cost $c \in (0, \bar{c}]$ with $\bar{c}$ the VOLL.
- Technology frontier: convex capital cost function $k(c)$

- Infinitely many small firms can invest in technology $c$
- Total expected profit: $\pi(c) = T(b(c)) - k(c) - c \cdot H(b(c))$:
  - expected transfers: $T(b(c))$,
  - expected capacity factor: $H(b(c))$. 
Model Set-up: Demand

- Consumers are price takers.
- Stochastic and elastic inverse demand function: \( p = P(q) + \varepsilon \).
  Normalized such that \( \varepsilon \) is the intercept of the demand function \( (P(0) = 0) \).
- Demand shock \( \varepsilon \) distributed with CDF \( F(\varepsilon) \) over \([\underline{\varepsilon}, \bar{\varepsilon}]\).
  Quantile function \( Q(\cdot) = F^{-1}(\cdot) \).
Model Set-up: Market Clearing

- **Bidding and investment strategies** \{\(b(c), G(c)\)}:
  - \(b(c)\) bids by firm with marginal cost \(c\). Assume \(b'(c) > 0\).
  - \(G(c)\) total installed capacity with marginal costs equal or less than \(c\).
- **Market clearing** then determines clearing price \(p^{\varepsilon}\), quantity \(Q^{\varepsilon}\), and marginal power plant \(c^{\varepsilon}\) for any given demand shock \(\varepsilon\).
Model Set-up: Market Clearing

- We will index the different states of the world not by the demand shock $\varepsilon$ but by the marginal power plant $c$ (firm’s type).
- The **market clearing condition** when firm of type $c$ is marginal is

$$p(c) = b(c) = P(G(c)) + \varepsilon(c),$$

This determines $\varepsilon(c)$, the demand shock for which firm of type $c$ is marginal.
- The **capacity factor** $h(c)$ of a firm of type $c$ is then given by

$$h(c) = 1 - F(\varepsilon(c)).$$

- The **expected revenue** $T(c)$ of a firm of type $c$ under uniform price and pay-as-bid auctions:

$$T^{up}(c) = \int_{c}^{\bar{c}} b(t) dh(t), \quad T^{pab}(c) = b(c) h(c).$$
What constitutes a competitive bidding and investment equilibrium \( \{b(c), G(c)\} \)?

**Assumptions:**
- Producers invest and bid before the demand shock is realized (long-lasting bids).
- Producers are price-takers: they take the stochastic distribution of prices as given.
- No entry barriers.

**Competitive Market Equilibrium:**
- **Short-run:** firm sets \( b(c) \) to maximize profit for a given stochastic price distribution with CDF \( Z(p) \). This price distribution is consistent with market clearing:

\[
Z(p(c)) = F(\varepsilon(c)).
\]

- **Long-run:** firm makes zero expected profit \( \pi(c) = 0 \).
Analysis
I. Bidding Equilibrium

- The optimal bidding strategies follow the FOC (Federico & Rahman, 2003):

\[ b_{UP}(c) = c, \quad b_{PAB}(c) = c + \frac{1 - Z(b_{PAB}(c))}{Z'(b_{PAB}(c))}. \]

- PAB: trade-off between mark-up and being scheduled (similar to 1st price auction).
- However, the price distribution \( Z(p) \) is endogenous and depends on \( b(c) \).
- Hence, the optimal bid \( b(c) \) and the capacity factor \( h(c) \) are determined by a differential equation.
II. Investment Equilibrium

- Independent of auction format, from the envelope theorem the capacity factor $h(c)$ satisfies
  \[ h(c) = -k'(c). \]

- Intuition: Screening curves - which technology is the cheapest depends on capacity factor $h$ (Stoft, 2002; Boiteux, 1949). 

\[ h = \frac{-\Delta k}{\Delta c} = \frac{k^g - k^n}{c^g - c^n} \]
II. Investment Equilibrium (cont’d)

- Firm with technology $c$ bids its marginal cost (in UPA) or levelized cost (in PABA)

$$b^{UP}(c) = c, \quad b^{PAB}(c) = c + \frac{k(c)}{h(c)}.$$ 

- The Lerner index PABA is the reciprocal of the elasticity $\epsilon_k(c)$ of investment costs:

$$L = \frac{b(c) - c}{c} = \frac{k(c)}{|k'(c)|c} := \frac{1}{\epsilon_k(c)}.$$ 

Not due to market power, but necessary to recoup investment costs.

- The cumulative installed capacity $G(c)$ can be calculated from market clearing condition.
Example
A Functional-Form Model: Assumptions

- Linear demand function
  \[ P(q) = -\rho q \text{ with } \rho > 0. \]

- Convex investment cost
  \[ k(c) = \frac{1}{\gamma + 1} \frac{(\bar{c} - c)^{\gamma+1}}{\bar{c} - \underline{c}} \text{ with } \gamma \in (0, 1). \]

- Uniformly distributed demand shocks over \([\varepsilon, \bar{\varepsilon}]\).
Producers’ optimal bidding strategy and investment decision

Figure 2: Comparison of bidding (solid) and portfolios (dashed) between UPA and PABA.
Comparison of PABA versus UPA

- Investments
  - Aggregate investments are identical in the two auctions as in Fabra et al. (2011), $\bar{G}^{UP} = \bar{G}^{PAB}$.
  - But the generation mix is distorted.
  - Fewer investments in the baseload capacity ($G^{PAB} < G^{UP}$).
  - More investments in all intermediate technologies ($G^{PAB}' > G^{UP}'$).

- All firms make zero profit (free entry), so welfare $= CS$.
  - The UPA is efficient (= Peak-load pricing, Boiteux (1949)), so $CS^{UP} > CS^{PAB}$.
  - CS with high demand is higher: as volume is the same & consumers pay less.
  - CS with low demand is lower: as volume is smaller & price is higher.
Summary and Future Research
Summary of Results

- Our research speaks to the question how auction formats affect short-term (bidding) and long-term (investment incentives) decisions.

- Inefficiency does not necessarily originate from market power. It could come from market design. Under PABA,
  - In the short run, consumers' WTP is higher than producers' marginal costs.
    - Allocative inefficiency
  - In the long run, revenue for baseload is distorted downwards, and incentives for investment decrease.
    - Distortion in generation mix
Future Research

- Allow for some bunching, i.e. $G'(c) = 0$ for some $c$? Some intermediate technologies are not used.

- Introduce market power in a monopoly setting.
Thank you :)