

# Co-Movement between Commodity and Equity Markets Revisited - an Application of the Thick Pen Method

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# Background

- Increased co-movements/correlation
  - between equities and commodity returns
  - between non-energy and energy commodities
  - between index commodities > off-index
- Equilibrium levels of codependency
- Price information of commodities
- Investment decision

# This paper in a nutshell

- Return co-movement between commodity futures and equities
- Use of a novel approach called **Thick Pen Transform**
- Short-run and long-run components of co-movement
- Potential interest to investors, policy makers

# Data and Methodology

## Data

- Daily log-returns on prices of S&P500 Index
- Daily log-returns on prices of 22 commodity futures (nearby contracts)
- Period: 04 Jan 1993 - 24 Dec 2019 (6835)

## Model

- Thick Pen Measure of Association (TPMA) by [Fryzlewicz and Oh \(2011\)](#)
- Multi-thickness Thick Pen Measure of Association (MTTPMA) by [Jach \(2021\)](#)

# Why?

- Stationary and non-stationary
- More than two time series
- Time-evolving
- Given time scale
- Across different time scale
- Visually interpretable

## In comparison with traditional co-movement measures

- Lack the multi-frequency outlook (e.g. cross-correlation of multi-period, rolling-window cross correlation)
- Impose particular structure on the data-generating mechanism/ co-movement dynamics (e.g. wavelet-based metrics)
- Lack cross-term co-movement ( [Gorrostieta et al. 2019](#) )

# Thick Pen Transform (TPT)

Let,

- $X = (X_t)_{t=1}^T$  - a univariate time series;
- $\mathcal{T}$  - set of positive constant thickness parameters i.e.  $\tau_i \in \mathcal{T}, i = 1, 2, \dots, |\mathcal{T}|$  ( $|\mathcal{T}|$  is the cardinality of  $\mathcal{T}$ );
- lower boundaries of the area

$$L_t^\tau(X) = \min(X_t, X_{t-1}, \dots, X_{t-\tau})$$

- upper boundaries of the area

$$U_t^\tau(X) = \max(X_t, X_{t-1}, \dots, X_{t-\tau})$$

- A collection of  $n$  pairs of boundaries ( $2.n.T$  random variables in total) denoted as

$$TP_{\mathcal{T}}(X) = \{(L_t^\tau(X), U_t^\tau(X))_{t=1}^T\}_{\tau \in \mathcal{T}}$$

## Thick Pen Measure of Association (TPMA)

- Quantify co-movement between two/more time series for a given time scale

$$\rho_t^\tau(X^{(1)}, X^{(2)}, \dots, X^{(K)}) = \frac{\min_k(U_t^\tau(X^{(k)})) - \max_k(L_t^\tau(X^{(k)}))}{\max_k(U_t^\tau(X^{(k)})) - \min_k(L_t^\tau(X^{(k)}))}$$

## Multi-thickness Thick Pen Measure of Association (MTTPMA)

- Quantify co-movement between two/more time series for multi-time scale

$$\rho_t^{(\tau^{(1)}, \tau^{(2)}, \dots, \tau^{(K)})}(X^{(1)}, X^{(2)}, \dots, X^{(K)}) = \frac{\min_k(U_t^{\tau^{(k)}}(X^{(k)})) - \max_k(L_t^{\tau^{(k)}}(X^{(k)}))}{\max_k(U_t^{\tau^{(k)}}(X^{(k)})) - \min_k(L_t^{\tau^{(k)}}(X^{(k)}))}$$

- This measure is restricted to interval  $\rho_t^\tau(X^{(1)}, X^{(2)}) \in (-1, 1]$  and is time-varying.

# Empirical Results

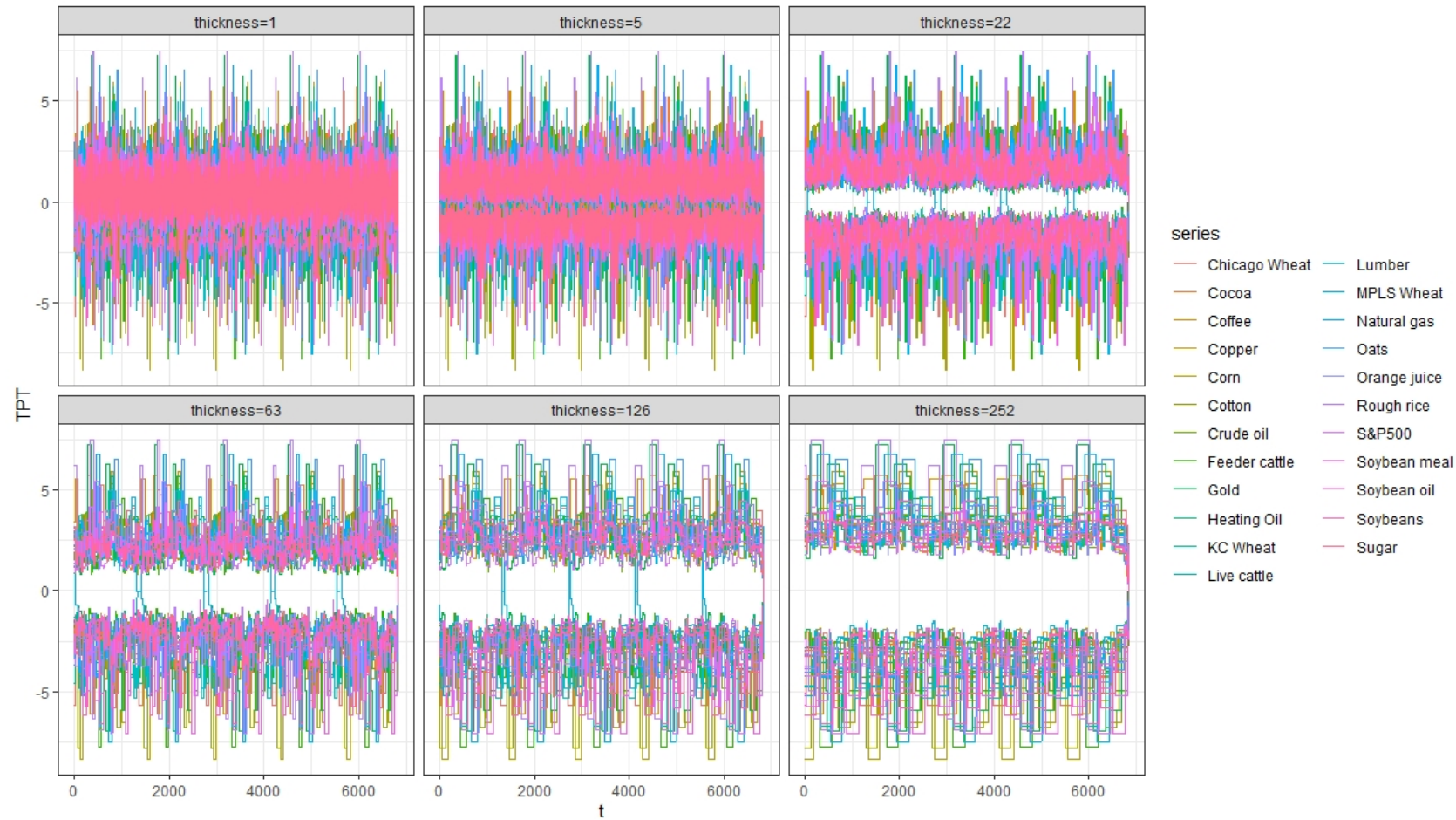


Figure 1: Thick Pen Transform daily returns (normalised) of equity index and commodity futures) for several thickness up to 252 trading days.

# Empirical Results

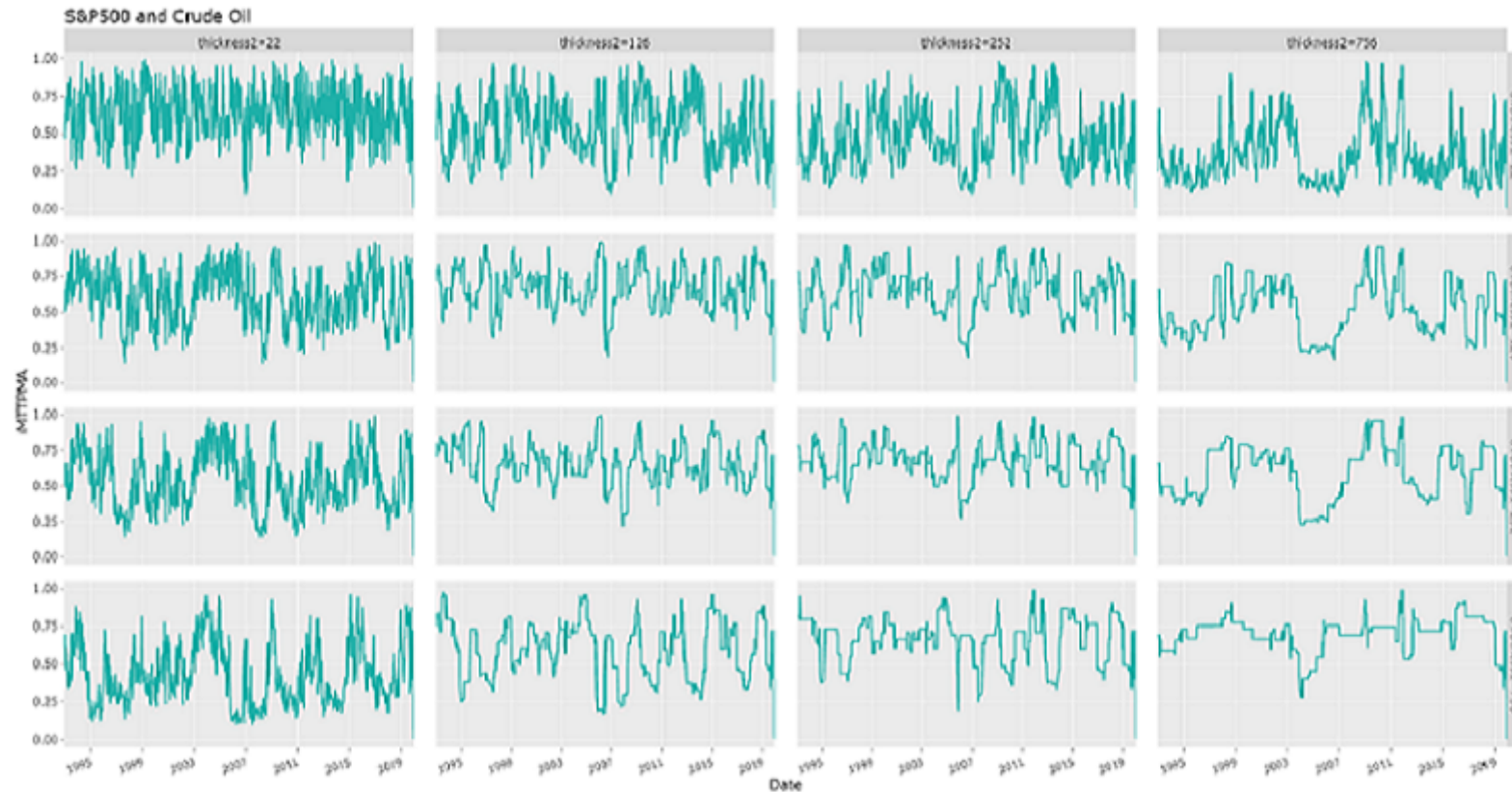


Figure 2: Multi-thickness Thick Pen Measure of Association of daily returns (normalised) of the equity index and crude oil futures (energy index). Thickness 22, 126, 252 and 756 represents short term (1-month, 6-month) and long-term (1-year, 3-year) component respectively.

# Empirical Results

TPMA of daily realised return of equity index and commodity futures

|                                | 6-month                |                | 3-year                 |                |
|--------------------------------|------------------------|----------------|------------------------|----------------|
|                                | Since financialisation | Changing point | Since financialisation | Changing point |
| <b>S&amp;P500-energy index</b> |                        |                |                        |                |
| Crude Oil                      | similar                | 2006/2007      | higher                 | 2004           |
| Heating Oil                    | higher                 | 2004           | higher                 | 2004           |
| Natural Gas                    | similar                | 2006/2007      | higher                 | 2004           |
| <b>S&amp;P500-off-index</b>    |                        |                |                        |                |
| Minneapolis Wheat              | lower                  | 2008           | lower                  | 2004/2005      |
| Soybean Meal                   | lower                  | 2004           | lower                  | 2004 and 2011  |
| Rough Rice                     | higher                 | 2004           | higher                 | 2004           |
| Oats                           | lower                  | 2004           | higher                 | 2004 and 2012  |
| Orange Juice                   | higher                 | 2008           | higher                 | 2004           |
| Lumber                         | higher                 | 2004           | similar                | 2004           |

# Empirical Results (Continued)

TPMA of daily realised return of equity index and commodity futures

|                                    | 6-month                |                | 3-year                 |                |
|------------------------------------|------------------------|----------------|------------------------|----------------|
|                                    | Since financialisation | Changing point | Since financialisation | Changing point |
| <b>S&amp;P500-non-energy index</b> |                        |                |                        |                |
| Chicago wheat                      | higher                 | 2004/2005      | higher                 | 2003/2004      |
| Kansas City Wheat                  | higher                 | 2004/2005      | higher                 | 2007/2008      |
| Corn                               | higher                 | 2008           | similar                | 2011           |
| Soybeans                           | higher                 | 2004           | similar                | 2004           |
| Soybean Oil                        | lower                  | 2004/2005      | lower                  | 2004           |
| Coffee                             | higher                 | 2004           | similar                | 2004           |
| Sugar                              | higher                 | 2008           | similar                | 2004           |
| Cocoa                              | higher                 | 2004           | higher                 | 2004           |
| Cotton                             | higher                 | 2004           | higher                 | 2004           |
| Live Cattle                        | similar                | 2004           | lower                  | 2004           |
| Feeder Cattle                      | lower                  | 2004           | lower                  | 2003/2004      |
| Copper                             | higher                 | 2004           | higher                 | 2004           |
| Gold                               | similar                | 2007           | similar                | 2003/2004      |

# Key Findings

- Overall, increased association between equity and commodity futures
- Co-dependency between equity and off-index futures are lower than with other commodities (both SR & LR)
- Energy index futures and equities co-move to a larger extent since financialisation (LR)
- Weak co-movement between the equity index and soybean based and livestock futures (both SR & LR)
- Evidence of asymmetric effects in cross-comovement i.e. MTTPMA

# Conclusions

- Promising technique to quantify cross-dependency
- Short-term and long-term policy perspectives
- Uncertainty caused by energy transition → structural change ( [Fattouh et al. 2018](#) )
- Formulate trading strategy
- Energy transition

# Working Paper

- Co-movement between commodity and equity markets revisited - an application of the Thick Pen method (with M. Gronwald, R. B. Durand, and S. Lee) USAEE/IAEE Working Paper Series No. 21-521 (2021)

# Thanks!

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