When are reserves needed? Co-optimization of energy and reserves in electricity markets with multiple technologies

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Motivation

- Due to unforeseen demand and system load fluctuations, many electricity market elements are designed to provide electricity reliably. Those markets provide products like energy (SR), reserves (SR), capacity (LR), etc.
- With so many elements, we are asked one question: energy and reserves are both purchased for continuously matching the demand and supply. Why do we need both?
- Energy and reserves are kind of substitutes, so why can't we co-optimize them to minimize cost and signal price of each.

Related Literature

1. Why reserves?
   - Reserves are a way to provide reliability.
     Joskow and Tirole (2007), Wilson (2002), Cramton (2017)
   - Reserves are financial hedges.
     Kleindorfer and Wu (2005), Anderson et al (2017)
2. How does reserves market work?
   - Auction design in reserves market (to reveal private cost)
     Bushnell and Oren (1994), Chao and Wilson (2002)
3. Co-optimization (more in an engineering way)
   - Tan & Kirschen (2006), Hassan et.al. (2018)

Contribution

1. I propose a simple economic model in which energy and reserves are co-optimized.
2. I give another reason for the existence of reserves.
3. I give state contingent dispatch load of both reserves and energy.
4. Efficient technology frontier in which energy and reserves are produced is provided.

Preview of Main Results

- Reserves and energy can be regarded as two types of call options.
- Reserves are not necessary. There is a threshold of standby cost k^{FB}, such that reserves are (not) procured if the standby cost is smaller (larger) than the threshold.
- Increasing usage of renewables appeals for attention to reserves.
- There exists a convex line such that only technologies lying on this line will be used to provide energy or reserves.

1 Introduction

2 **Model**

3 Supply Uncertainty

4 Conclusion

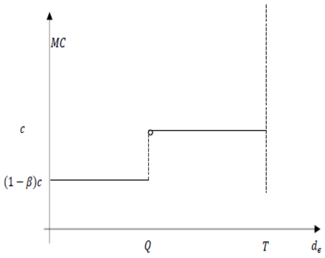
Model Setup: Supply Side

- n technologies from the technology set $N = \{1, 2, ..., n\}$ are available to produce one type of **non-storable** good in two ways:
  - Energy:
    - for any technology i, unit production cost c_i, β_i : sunk cost ratio.
    - suppose quantity of energy is Q_i, but only q_i is dispatched. Then the total cost of energy is $c_i Q_i - (1-\beta_i) c_i (Q_i - q_i)$. That is, energy production can be reduced with part of production cost recouped.
  - Reserves:
    - unit incur standby cost k_i, and unit production cost c_i for **activated** reserves;
    - suppose quantity of reserves is R_i, and quantity of activated reserves is $0 \leq r_i \leq R_i$, the total cost of reserves is $k_i R_i + c_i r_i$.
- $Q = \sum_{i=1}^{n} Q_i, R = \sum_{i=1}^{n} R_i$. Total supply $T = \sum_{i \in N} (Q_i + R_i)$.

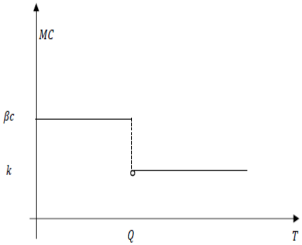
Introduction

Crux of Supply Side:

1. Energy and reserves have similar cost structure but different in amount of costs:
    - sunk cost × energy quantity + incremental production cost × dispatched energy
    - standby cost × reserves quantity + production cost × activated reserves
2. They both behave like call options and can be used as substitutes.



(a) production cost



(b) option cost

Figure: Cost Structure

Model Setup: Demand Side

- Planner Optimization.
- Homogeneous consumers and price takers;
- Stochastic demand $D = \varepsilon$ distributes from $[0,1]$, pdf f_ε, cdf F_ε.
- Demand is inelastic up to the reservation price u_ε. $u'_\varepsilon \geq 0, u''_\varepsilon \leq 0$

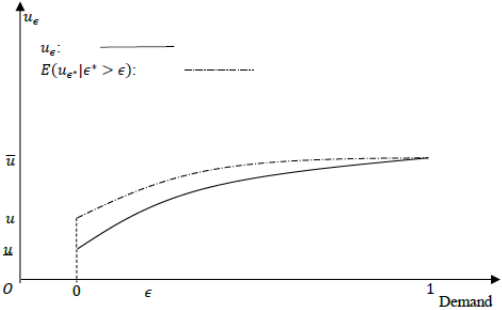


Figure: willingness to pay

Model Setup: Surplus

- It is possible that market does not clear:
  1. **Shortage of Supply:** demand ε is larger than planned supply T;
  2. **Rationing:** in some states ε, it is cost effective not to dispatch all available load.
- Denote the dispatched load as $d_\varepsilon \leq \min\{T, \varepsilon\}$, Total gross surplus:

$$S(\varepsilon | T, Q) = d_\varepsilon \cdot u_\varepsilon \tag{1}$$

Timeline (Two-stage)

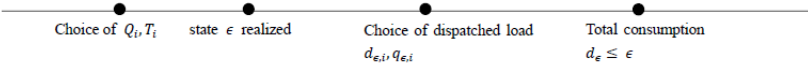


Figure: Timeline

Ex-ante Expectation

- Ex-ante expected utility of consumers $U(T)$:

$$U(d_\varepsilon) = \mathbb{E}(S) = \int_0^1 (u_\varepsilon d_\varepsilon) dF_\varepsilon \tag{2}$$

- Expected total cost:

$$\begin{aligned} & C(\{Q_i\}, \{T_i\}, \{d_{\varepsilon,i}\}, \{q_{\varepsilon,i}\}) \\ & = \sum_{i \in N} \int_0^1 \left((1-\beta_i) c_i q_{\varepsilon,i} + \beta_{\sigma_j} c_i Q_i \right) dF_\varepsilon \\ & + \sum_{i \in N} \int_0^1 \left[c_i (d_{\varepsilon,i} - q_{\varepsilon,i}] + k_i (T_i - Q_i) \right) dF_\varepsilon \end{aligned} \tag{3}$$

Single Technology

Suppose there is only one available technology, the social net surplus maximization problem is:

$$\begin{aligned}
&\max_{Q,T,d_\varepsilon,q_\varepsilon} U(\{d_\varepsilon\},\{q_\varepsilon\}) - C(Q,T) \\
&\text{s.t. for all states } \varepsilon \in [0,1]: \\
&q_\varepsilon \geq 0 \\
&q_\varepsilon \leq d_\varepsilon \\
&q_\varepsilon \leq Q, (\gamma_\varepsilon) \\
&Q \leq T \\
&d_\varepsilon \leq \varepsilon, (\lambda_\varepsilon) \\
&d_\varepsilon \leq T, (\mu_\varepsilon)
\end{aligned} \tag{4}$$

where λ_ε, γ_ε and μ_ε are the shadow prices of constraints.
μ_ε: shadow price of reserves; $\mu_\varepsilon + \gamma_\varepsilon$: shadow price of energy.

Proposition 1 (second-stage)

Proposition

With perfect information, given any c and β, the efficient dispatching in different states show that:

a. *when* $u_\varepsilon < (1-\beta)c$,

$$d_\varepsilon = 0, \mu_\varepsilon = \gamma_\varepsilon = 0; \tag{5}$$

b. *when* $(1-\beta)c \leq u_\varepsilon < c$,

$$d_\varepsilon = q_\varepsilon = \min\{Q, \varepsilon\} \tag{6}$$

$$\mu_\varepsilon + \gamma_\varepsilon = \begin{cases} 0, & Q > \varepsilon \\ u_\varepsilon - (1-\beta)c, & \varepsilon \geq Q \end{cases} \tag{7}$$

c. *when* $u_\varepsilon \geq c$,

$$d_\varepsilon = \min\{T, \varepsilon\} \tag{8}$$

$$\gamma_\varepsilon = \begin{cases} \beta c, & Q \leq \varepsilon \\ 0, & Q > \varepsilon \end{cases} \tag{9}$$

$$\mu_\varepsilon = \begin{cases} u_\varepsilon - c, & T \leq \varepsilon \\ 0, & T > \varepsilon \end{cases} \tag{10}$$

Shadow Price

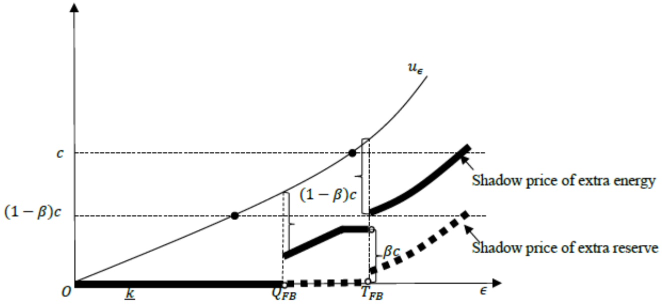


Figure: Shadow Price of Energy and Reserves

Proposition 1 (first stage)

Proposition

With perfect information, there exists a unique threshold $k^{FB}(c)$ *such that for* $k \geq k^{FB}(c)$, $R_{FB} = 0$, $T_{FB} = Q_{FB}$. *That is, only energy is available, and its amount is uniquely solved by:*

$$\int_0^1 1(\varepsilon > Q_{FB}) \max\{u_\varepsilon - (1-\beta)c, 0\} dF_\varepsilon = \beta c \tag{11}$$

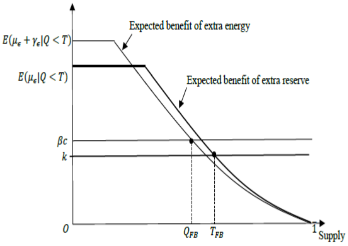
When $k < k^{FB}(c)$, $R_{FB} > 0$, *the principal will procure both energy and reserves,* $R_{FB} = T_{FB} - Q_{FB}$. *The first-best total supply* T_{FB} *is uniquely characterized by:*

$$\int_0^1 1(\varepsilon > T_{FB}) \max\{u_\varepsilon - c, 0\} dF_\varepsilon = k \tag{12}$$

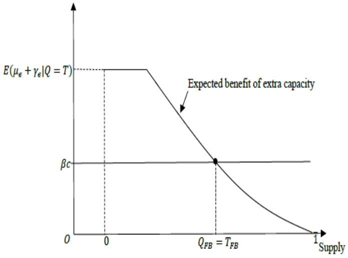
and the first-best energy supply Q_{FB} *is determined by:*

$$\int_0^1 1(\varepsilon > Q_{FB}) \max\{u_\varepsilon - (1-\beta)c, 0\} dF_\varepsilon - \beta c = \int_0^1 1(\varepsilon > Q_{FB}) \max\{u_\varepsilon - c, 0\} dF_\varepsilon - k \tag{13}$$

Figure: Determination of Supply



(a) $k < k^{FB}$



(b) $k > k^{FB}$

Figure: Determination of Supply

Multi-Technology

Proposition

When there are n technologies, $i \in N = \{1, 2, \ldots n\}$ with observable production cost c_i, sunk cost ratio β_i and standby cost k_i, there exist a subset of technologies used for energy, $S \subseteq N$, which order follows σ, and can be normalized to σ^S, and a subset of technologies used for reserves, $X \subseteq N$, which follows cardinal order, and can be normalized to σ^X. When $X \neq \emptyset$, the first-best total supply T_{FB} is uniquely characterized by:

$$\int_0^1 1(\varepsilon > T_{FB}) \max\{u_\varepsilon - c_{\sigma^X(|X|)}, 0\} dF_\varepsilon = k_{\sigma^X(|X|)} \tag{14}$$

and the first best energy supply Q_{FB} is determined by:

$$\int_0^1 1(\varepsilon > Q_{FB}) \max\{u_\varepsilon - (1 - \beta_{\sigma^S(|S|)}) c_{\sigma^S(|S|)}, 0\} dF_\varepsilon - \beta_{\sigma^S(|S|)} c_{\sigma^S(|S|)}$$
$$= \int_0^1 1(\varepsilon > Q_{FB}) \max\{u_\varepsilon - c_{\sigma^X(1)}, 0\} dF_\varepsilon - k_{\sigma^X(1)}$$
(15)

Proposition 2 (Part B)

Proposition

$R^{FB} = T^{FB} - Q^{FB}$. *The allocation of reserves among technologies is given by:*

$$\text{for } x \in \{1,2,3,...,|X|-1\}, \tag{16}$$

$$\int_0^1 1(\varepsilon > Q_{FB} + \sum_{t=1}^{x} R^{FB}_{\sigma^X(t)}) \max\{u_\varepsilon - c_{\sigma^X(t)}, 0\} dF_\varepsilon - k_{\sigma^X(t)}$$
$$= \int_0^1 1(\varepsilon > Q_{FB} + \sum_{t=1}^{x} R^{FB}_{\sigma^X(t)}) \max\{u_\varepsilon - c_{\sigma^X(t+1)}, 0\} dF_\varepsilon - k_{\sigma^X(t+2)} \tag{17}$$

$$R^{FB}_{\sigma^X(|X|)} = R^{FB} - \sum_{x \in X \setminus \{\sigma^X(|X|)\}} R^{FB}_{\sigma^X(x)} \tag{18}$$

The allocation of energy mong technologies is given by:

$$\text{for } s \in \{1,2,3,...,|S|-1\}, \tag{19}$$

$$\int_0^1 1(\varepsilon > \sum_{t=1}^{s} Q^{FB}_{\sigma^S(t)}) \max\{u_\varepsilon - (1-\beta_{\sigma^S(t)}) c_{\sigma^S(t)}, 0\} dF_\varepsilon - \beta_{\sigma^S(t)} c_{\sigma^S(t)}$$
$$= \int_0^1 1(\varepsilon > \sum_{t=1}^{s} Q^{FB}_{\sigma^S(t)}) \max\{u_\varepsilon - (1-\beta_{\sigma^S(t+1)}) c_{\sigma^S(t+1)}, 0\} dF_\varepsilon - \beta_{\sigma^S(t+1)} c_{\sigma^S(t+1)} \tag{20}$$

$$Q^{FB}_{\sigma^S(|S|)} = Q^{FB} - \sum_{s \in S \setminus \{\sigma^S(|S|)\}} Q^{FB}_{\sigma^S(s)} \tag{21}$$

Figure: Efficient Technology Frontier (No Rationing)

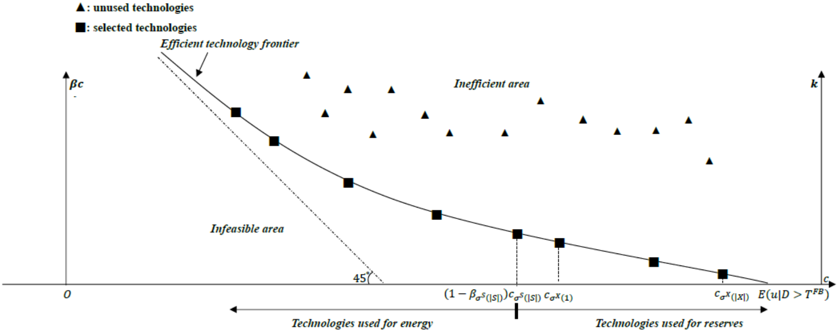


Figure: Efficient Technology Frontier

New set-up

Now, we add supply uncertainty as an uncertain capacity availability factor ϕ_i. Hence, the stochasticity is modeled as $(\varepsilon, \{\phi\})$, which follow a distribution $f(\varepsilon, \{\phi_i\})$.

To give an insight into how renewables integration affect our results, we use a simple model:

- There are two technologies: renewables (low production cost, effective curtailment, large upward inflexibility) and traditional technology (high production cost, high flexibility). $c_1 < c_2, \beta_1 < \beta_2, k_1 \gg k_2$.
- Only the availability of renewables $\phi \in [0,1]$ is stochastic. The availability factor of the traditional technology is equal to 1.
- $\frac{\partial F(\varepsilon|\phi)}{\partial \phi} > 0$.
- the willingness to pay $u_{\varepsilon,\phi} = u > \max\{c_i\}$.

Results

$$\int_0^1 (u - c_2)[1 - F(T - (1-\phi)Q|\phi)]dF(\phi) - k_2 = 0 \tag{22}$$

$$\int_0^1 (u - c_2)[1 - F(\phi Q|\phi)]dF(\phi) - k_2 = \int_0^1 (u - (1-\beta_1)c_1)[1 - F(\phi Q|\phi)]\phi dF(\phi) - \beta_1 c_1 > 0 \tag{23}$$

- $T' > T$: the total supply should be larger;
- $R' > 0$: reserves are necessary if $E(\phi)$ is small.

Figure: Integration of Renewables

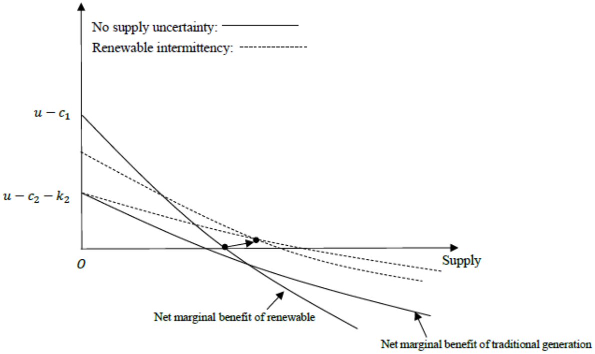


Figure: Integration of Renewables

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3 Supply Uncertainty

4 **Conclusion**

Implications

- Without supply uncertainty, reserves are not always cost-efficient to provide flexibility. The conditions are low standby cost, or high willingness to pay, or both.
- However, when intermittent renewables are integrated, reserves become more important.
- Not all technologies are efficient to provide energy and/or reserves. The selection of technologies depend on cost trade-off and the uncertainties we would consider.

Thank you!