Nord Pool System Price: Nonlinear Error-Shock Analysis

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Nord Pool System Price: Nonlinear Error - Shock Analysis

1. Introduction

A new and extended study of the dynamics of the Nordic/Baltic System Price:

- auction market with aggregate demand and supply balance daily prices

- market equilibrium includes supply surpluses (low and even negative prices) and demand shortages (capacity constraints/high prices).
  - high demand and capacity constraints suggest a vertical supply curve (shocks)
  - low demand and supply surplus (much wind) suggest a close to horizontal (forced; zero/negative prices) supply curve (shocks)

- Energy facility production stops (hydro/nuclear) for maintenance or failures (not announced in the OPM system) suggest a surprise supply curve shift to the left (shocks)

- An extensive increase in wind energy investments has now created a more volatile energy production system giving spur to both negative and positive capacity constraints and a challenge to the Nordic/Baltic energy grid (shocks)

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Nord Pool System Price: Nonlinear Error - Shock Analysis

1. Introduction

An extended study of the dynamics of the Nordic/Baltic System Price:

- deterministic factors (i.e. seasonal factors) are present in prices
  - in the price change process
  - in the volatility process

- the price process possess an unknown distributional density which are far from normally distributed

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2. Non-linear Error-Shock Methodology

Gallant et al. (1993) defines the one-step ahead mean conditioned on all history as:

$$g(y_{t-\infty+1},\dots,y_t) = E\left(y_{t+1} / (y_{t-k})_{k=0}^{\infty}\right)$$

Markovian:

$$g(y_{t-L+1},\dots,y_t) = E\left(y_{t+1} / (y_{t-k})_{k=0}^{L-1}\right)$$

Setting :

$$\hat{y}_j(x) = E\left(g\left(y_{t-L+j},\dots,y_{t+j}\right) / x_t = x\right)$$
$$= E\left(E\left(y_{t+j} / y_{t-L+j},\dots,y_{t+j}\right) / x_t = x\right)$$

we obtain $\hat{y}_j^{\,i}$ for i = -20%,…., 0,…, 20%, j=1, 2,…, 5,
where $x = (y_{-L+1},\dots,y_0)$

This is the conditional expectations of the trajectories of the one-step ahead conditional mean: $\hat{f}(y|x)$.

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2. Non-linear Error-Shock Methodology

Gallant et al. (1993) defines the one-step ahead variance as the one-step ahead forecast of the variance conditioned on all history:

$$Var\left(y_{t+1} / (y_{t-k})_{k=0}^{\infty}\right) = E\left\{\left[y_{t+1} - E\left(y_{t+1}\{y_{t-k}\}_{k=0}^{\infty}\right)\right] x \left[y_{t+1} - E\left(y_{t+1} / \{y_{t-k}\}_{k=0}^{\infty}\right)\right]` / \{y_{t-k}\}_{k=0}^{\infty}\right\}$$

Markovian: $Var\left(y_{t+1} / (y_{t-k})_{k=0}^{L-1}\right)$

Defining the function *g(.)*, we can measure shocks on volatility. Writing:

$$\psi_j(x) = E\left(g\left(y_{t-L+j},...,y_{t+j}\right) / x_t = x\right)$$
$$= E\left(\mathrm{Var}\left(y_{t+j} / y_{t-L+j},...,y_{t+j}\right) / x_t = x\right)$$

for $j = 1, 2, \ldots,$ and where $x = \left(y_{-L+1},...,y_0\right)$.

This is the conditional expectations of the trajectories of the one-step ahead conditional variance matrix *j* steps ahead, conditioned on $x_t = x$, that is $\hat{f}\left(y \mid x\right)$.

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2. Non-linear Error-Shock Methodology

Analytical evaluation of the integrals are intractable. However, Monte Carlo simulation is well suited.

Therefore let $\left\{ y_j^r \right\}_{j=1}^{\infty}$ where $r = 1, 2, \ldots, R$, be R simulated simulations of the process starting from $x_0 = x$.

y_1^r is a random drawing from $f(y \mid x)$ with $x = \left(y'_{-L+1}, \ldots, y'_{-1}, y'_0 \right)'$; y_2^r is a random drawing from $f(y \mid x)$ with $x = \left(y'_{-L+2}, \ldots, y'_0, y'_1 \right)'$; and so forth.

Applying the invariant function of a stretch of $\left\{ y_j \right\}$ of length $J+1$, we get

$$\bar{g}_j(x) = \int \ldots \int g\left(y_{j-J}, \ldots, y_j\right) \left[\prod_{i=0}^{j-1} f\left(y_{i+1} / y_{y-L+1}, \ldots, y_i\right) \right] dy_1 \ldots dy_j$$

$$\approx (1/R) \sum_{r=1}^{R} g\left(y_{j-J}^r, \ldots, y_j^r\right)$$

with the approximation error tending to zero almost surely as $R \to \infty$, under mild regulatory conditions on f and g.

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2. Non-linear Error-Shock Methodology

For statistical inference, sup-norm bands are constructed by bootstrapping, using simulations to take into account the sampling variation in the estimation of $\hat{f}(y \mid x)$

That is, changing the seed that generates densities and impulse response samples.

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3. Literature review

3.1 The Electricity Nord Pool Spot System Prices

Early empirical references:

Solibakke (2002).
Goto and Karolyi (2004)
Higgs and Worthington (2005 and 2008)
Knittel and Roberts (2005)
Chan and Gray (2006)
Theodorou and Karyampas (2008)

+ several newer applications

Stochastic modelling of commodities:

Lucia and Schwartz, 2002 and
Geman and Roncoroni, 2006

+ several newer applications

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Nord Pool System Price: Nonlinear Error-Shock Analysis

3. Literature review
3.1 The Electricity Nord Pool Spot System Prices

Main Findings:

Deterministic factors with strong seasonality (day of the week, weekends, month, and summer/winter) abrupt jumps both up and down and high and changing volatility.

Moreover, mean-reversion effects with seasonal changes in volatilities as well as volatility clustering

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3. Literature review
3.1 Nonparametric model expansion methodology (*SNP*)

Maximum likelihood estimates of the conditional mean and volatility (simultaneous parameter estimation).

References:
Bollerslev (1986),
Bollerlev et al. (1987, 1992)
Engle et al. (1986, 1993)
Nelson (1991)
deLima (1995a, 1995b)
Gouriéroux (1997)

SNP (Semi NonParametric) estimation:

Main References:
Gallant et al. (1987, 1992, and 1996) (www.aronaldg.org)

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3. Literature review
3.2 Nonparametric model expansion methodology (*SNP*)

SNP (Semi Non-Parametric) estimation starts with an established parametric model; higher order terms capture departures from this model

- does not suffer from the course of dimensionality
- where data are sparse, the leading term fill in smoothly between data points
- where data are plentiful, the higher order terms accommodate deviations from the leading term and fits are compared to kernel estimates
- hermite series expansion (attractive both for modelling and computations)
- these models are generally considered to give excellent first approximations in a variety of applications
- hermite densities are easy to evaluate, differentiate and moments are easily evaluated
- hermite densities are practical for simulations

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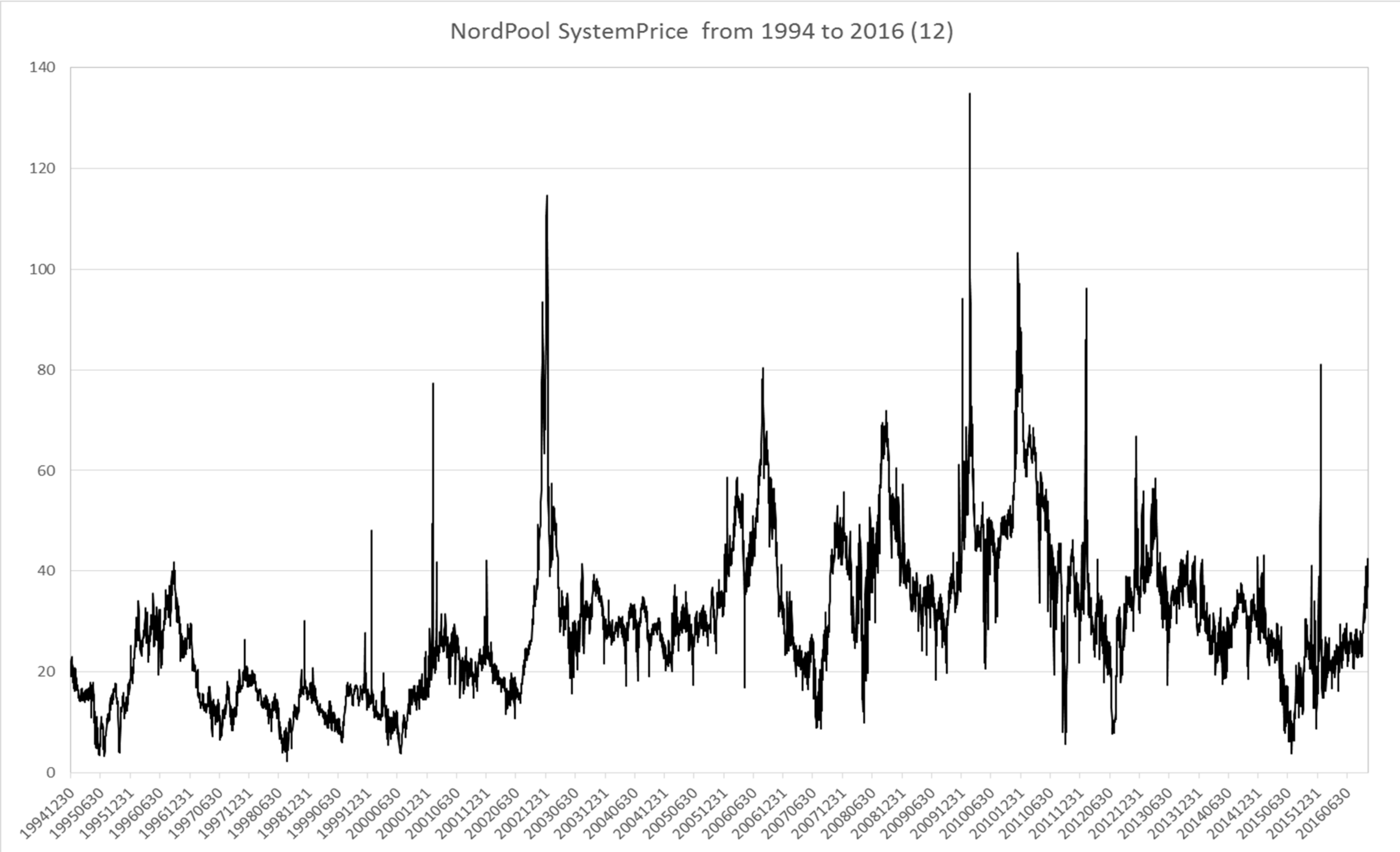
4. Nord Pool System Price Data for the period 1994 – 2016 (11)



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4. Nord Pool System Price and Adjustments for Deterministic Factors

Let ϖ denote the System Price series to be adjusted by the procedure. Initially, we fit the mean regression $\varpi = x \cdot \beta + u$, where x consists of systematic calendar, time of year, weekends, holidays, school and public holidays etc. as are most convenient for the time series (5 – 7 countries). Moreover, x contains parameters for linear trends, squared trends, and separation variables.

The residuals from this regression are denoted $\hat{u}$.

The variance equation model now becomes: $\hat{u}^2 = x \cdot \gamma + \varepsilon$ and is estimated.

We now form: $z = \frac{\hat{u}^2}{\sqrt{e^{x \cdot \hat{\gamma}}}}$, giving a series with mean zero and unit (approx.) variance.

Finally, $\hat{\varpi} = a + b \cdot (\frac{\hat{u}}{\sqrt{e^{z \cdot \hat{\gamma}}}})$ is the adjusted series, where a and b is chosen so that

Mean: $\frac{1}{T} \cdot \sum_{i=1}^{T} \hat{\varpi}_i = \frac{1}{T} \cdot \sum_{i=1}^{T} \varpi_i$ and Var: $\frac{1}{T-1} \cdot \sum_{i=1}^{T} (\hat{\varpi}_i - \overline{\varpi})^2 = \frac{1}{T-1} \cdot \sum_{i=1}^{T} (\hat{u}_i - \overline{u})^2$

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Nord Pool System Price: Nonlinear Error-Shock Analysis

4. Nord Pool System Price and Adjustments for Deterministic Factors

Table 1. Characteristics of Returns and Volatility Raw System Price Series

Returns ($\ln(S_t / S_{t-1})$)			Volatility ($\ln(res^2)$)		
Var	Coeff	t-Stat	Var	Coeff	t-Stat
INTR	-0.1684	0.6124	INTR	2.4967	25.9252
MAN	12.9491	33.6906	MAN	1.7222	20.6864
TUE	0.7755	2.0177	TUE	0.2745	3.2974
THR	-0.7717	2.0074	SAT	0.9200	11.0507
FRI	-1.8459	4.8013	SUN	0.3732	4.4829
SAT	-7.1302	18.5481	UKE3	-0.3079	1.5309
SUN	-3.5594	9.2597	UKE4	-0.5268	2.6194
UKE1	2.3361	2.3686	UKE6	-0.3169	1.5758
UKE18	-1.2436	1.6719	UKE7	-0.5154	2.5631
UKE26	1.9439	2.6133	UKE8	-0.5417	2.6936
UKE29	-1.3612	1.8300	UKE9	-0.4505	2.2402
UKE31	1.1968	1.6089	UKE10	-0.8432	4.1930
UKE32	1.4137	1.9005	UKE11	-0.9852	4.8987
UKE33	1.3267	1.7666	UKE12	-0.8130	4.0428
UKE39	1.2172	1.5996	UKE13	-0.7717	3.8375
………………					
			TREND	-3.0232	7.9974
			TREND2	3.3545	9.1654

Exogenous Variables (only close to significant coefficients are reported):
INTR=Constant; TUE=Thusday; WED=Wednesday; THR=Thursday; FRI=Friday, SAT=Saturday; SUN=Sunday; EASTRN=Eastern; OTHR=Other Holidays Week-1-Week-52; TREND=linear trend; TREND2=Squared trend

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4. Nord Pool System Price and Adjustments for Deterministic Factors

Estimation Comments:

Days and the mean: Mondays significant positive price changes. In contrast, Saturdays and Sundays negative price changes.

Days and the latent volatility: Mondays, Saturdays and Sundays contribute significantly to the high volatility in prices for Nord Pool electricity.

Trends: No mean effects.

Trends are significant for the latent volatility in both linear and non-linear form.

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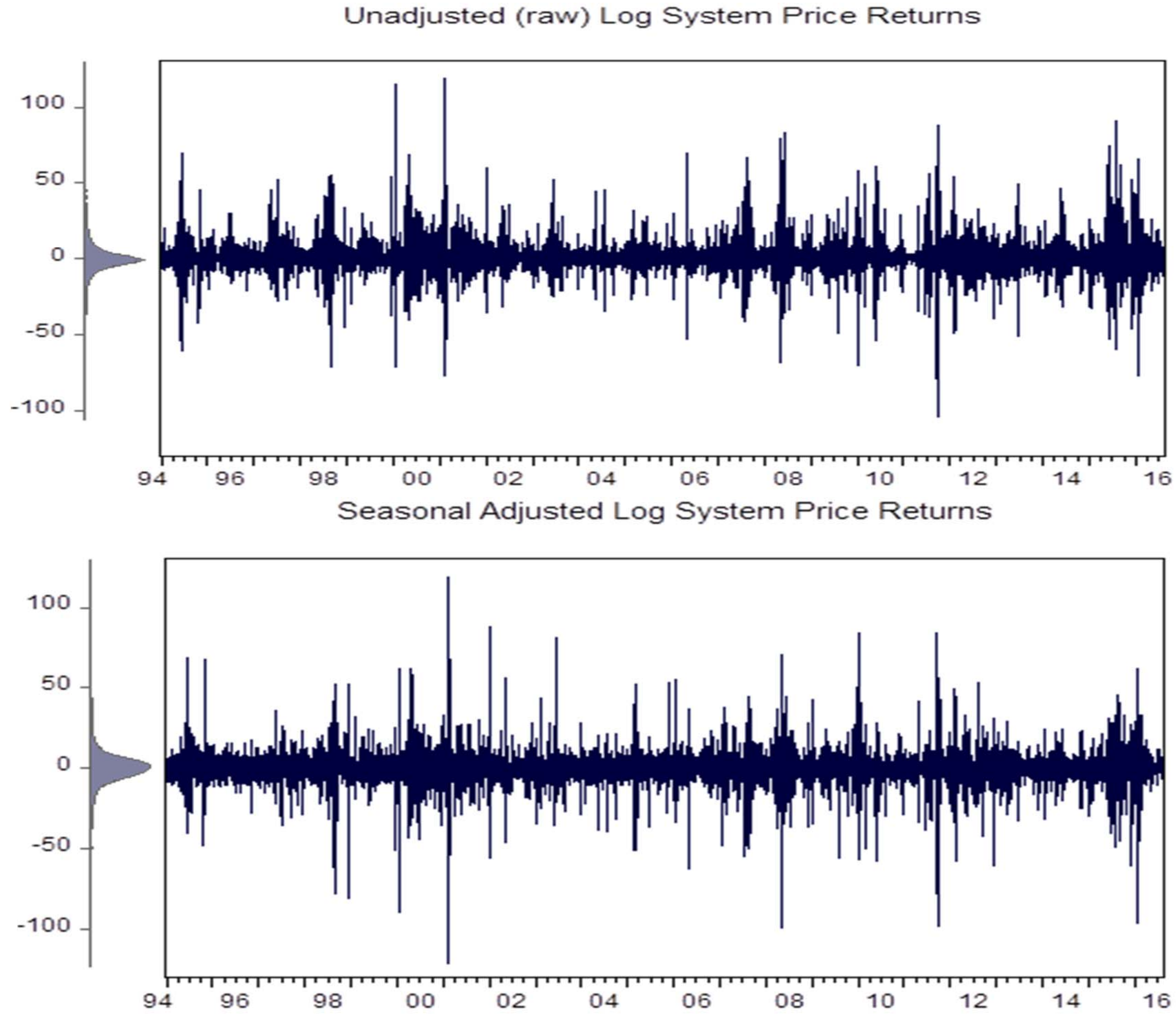
Nord Pool System Price: Nonlinear Error-Shock Analysis



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Nord Pool System Price: Nonlinear Error-Shock Analysis

4. Characteristics of the Nord Pool System Price Data

Table 2. Statistics for System Price Nordpool Spot Auction Market

Panel A: Unadjusted Series

Returns	Mean / Mode	Median Std.dev.	Maximum / Minimum	Moment Kurt/Skew	Quantile Kurt/Skew	Quantile Normal	Cramer-von-Mises	Serial dependence Q(12)	$Q^2(12)$
	0.00335	-0.60245	118.8913	14.75925	0.57813	114.1775	94.6867	2311.3	2703.80
		10.82066	-104.3984	0.84890	0.05678	{0.0000}	{0.0000}	{0.0000}	{0.0000}
	BDS-Z-statistic (e = 1)				KPSS (Stationary)		Augmented	ARCH	VaR 1% /
	m=2	m=3	m=4	m=5	Intercept	I + Trend	DF-test	(12)	CVaR 1%
	29.9551	38.2216	42.1053	45.3366	0.02821	0.00933	-19.6277	2244.70	-29.2708 %
	{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.6259}	{0.5219}	{0.0000}	{0.0000}	-43.6908 %

Panel B: Adjusted Series

Returns	Mean / Mode	Median Std.dev.	Maximum / Minimum	Moment Kurt/Skew	Quantile Kurt/Skew	Quantile Normal	Cramer-von-Mises	Serial dependence Q(12)	$Q^2(12)$
	0.00335	0.28816	119.4144	15.90260	0.09625	4.8494	51.6047	215.740	1327.70
		10.82066	-121.4442	-0.38241	-0.03701	{0.0885}	{0.0000}	{0.0000}	{0.0000}
	BDS-Z-statistic (e = 1)				KPSS (Stationary)		Augmented	ARCH	VaR 1% /
	m=2	m=3	m=4	m=5	Intercept	I + Trend	DF-test	(12)	CVaR 1%
	31.2438	37.1390	40.0875	42.1613	0.37611	0.03909	-26.6068	902.317	-32.8705 %
	{0.0000}	{0.0000}	{0.0000}	{0.0000}	{0.0846}	{0.1402}	{0.0000}	{0.0000}	-49.7361 %

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4. Nord Pool System Price Data and Adjustments for Deterministic Factors

Characteristics comments:

- Mean and standard deviation is unchanged (as announced)
- More normally distributed (but still far from normal)
- Lower serial correlation in both mean (Q) and volatility (Q^2 and ARCH)
- Strong data dependence (BDS-test statistic)
- Stationary series
- The VaR / CVaR risk measures have increased from the unadjusted to adjusted series.

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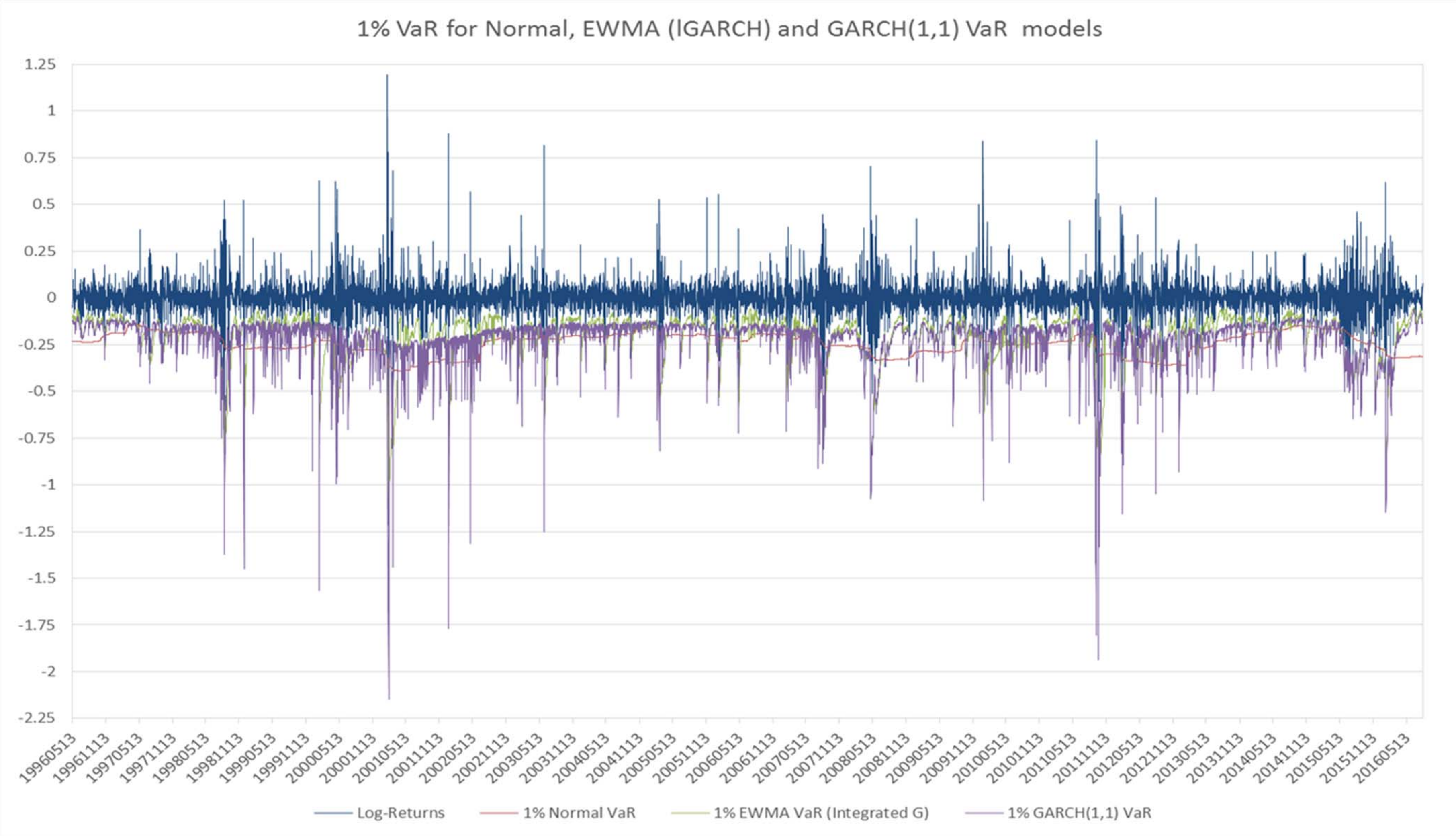
4. Nord Pool System Price Data and Adjustments for Deterministic Factors



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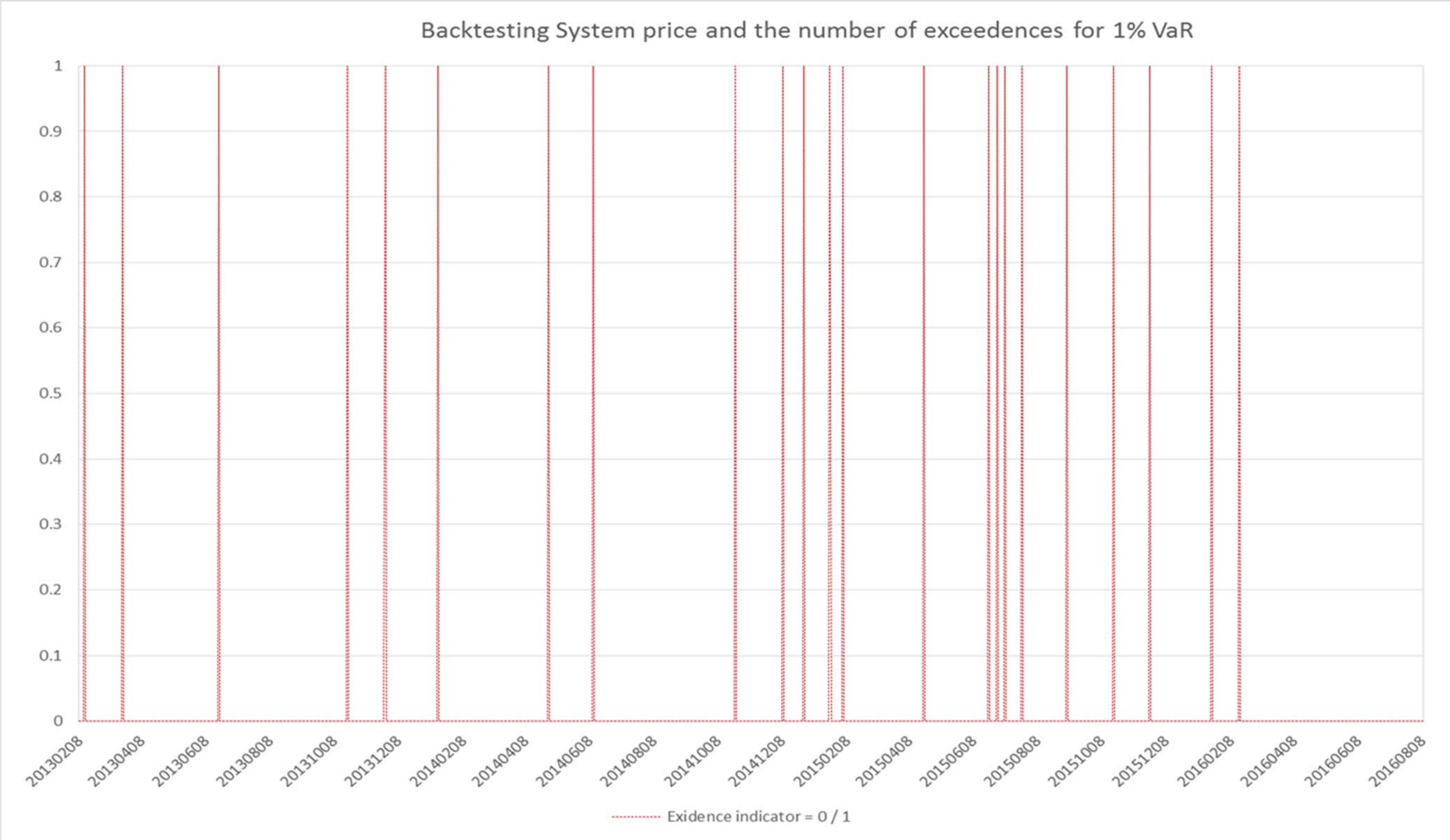
4. Nord Pool System Price Data and Adjustments for Deterministic Factors



LRCC = 7,94 ($\chi^2(1)$ distributed). *We cannot reject correct proportions of exceptions.*

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Nord Pool System Price: Nonlinear Error-Shock Analysis

5. The SNP model and Empirical Findings Non-linear Error-Shock Analysis
5.1 The SNP Density Projection ($f(y \mid x)$)

Classical Maximum Likelihood (*ML*) together with a *BIC* optimal (Schwarz, 1978) model selection strategy. The model building strategy support the model mean:

Table 3. NordPool Spot Electricity System Price

Statistical Model SNP-14,111,12, 000 -fit; semi-parametric-GARCH model

Mean Equation Var	SNP Coeff.	Mode	Standard error	t-statistics
Mean Correlation				
η_{13}	B(1,1)	0.00497	0.01267	0.39248
η_{14}	B(1,2)	-0.0691	0.01103	-6.26514
η_{15}	B(1,3)	-0.04054	0.01086	-3.73291
η_{16}	B(1,4)	-0.03263	0.01080	-3.02059
η_{17}	B(1,5)	-0.02738	0.01053	-2.59985
η_{18}	B(1,6)	0.02459	0.01024	2.40082
η_{19}	B(1,7)	0.12824	0.00979	13.09329
η_{20}	B(1,8)	0.01672	0.01004	1.66491
η_{21}	B(1,9)	-0.02091	0.00994	-2.10382
η_{22}	B(1,10)	-0.01319	0.00975	-1.35267
η_{23}	B(1,11)	-0.02629	0.00946	-2.7777
η_{24}	B(1,12)	-0.03391	0.00925	-3.66735
η_{25}	B(1,13)	0.00838	0.00898	0.93289
η_{26}	B(1,14)	0.07382	0.00874	8.44156

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Nord Pool System Price: Nonlinear Error-Shock Analysis

5. The SNP model and Empirical Findings Non-linear Error-Shock Analysis
5.1 The SNP Density Projection ($f(y \mid x)$)

The model building strategy support the model variance:

Table 3. NordPool Spot Electricity System Price

Statistical Model SNP-14,111,12, 000 -fit; semi-parametric-GARCH model

Mean Equation Var	SNP Coeff.	Mode	Standard error	t-statistics
Variance Equation				
η_{27}	R0[1]	0.23616	0.01455	16.22671
η_{28}	P[1,1]	0.40387	0.02869	14.07927
η_{29}	Q[1,1]	0.90472	0.00741	122.17024
η_{30}	V[1,1]	-0.33691	0.03783	-8.906
η_{31}	W[1,1]	0.69102	0.04225	16.35433

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5. The SNP model and Empirical Findings Non-linear Error-Shock Analysis
5.1 The SNP Density Projection (*f (y | x)*)

The model building strategy support these model exceptions from the parametric model:

Table 3. NordPool Spot Electricity System Price

Statistical Model SNP-14,111,12, 000 -fit; semi-parametric-GARCH model

Mean Equation			Standard	
Var	SNP Coeff.	Mode	error	t-statistics
Hermite Polynoms				
η_1	a0[1]	-0.00239	0.0059	-0.40531
η_2	a0[2]	-0.20835	0.00865	-24.08072
η_3	a0[3]	-0.02340	0.006	-3.90119
η_4	a0[4]	0.16943	0.00695	24.3652
η_5	a0[5]	0.00820	0.00674	1.21679
η_6	a0[6]	-0.08675	0.0081	-10.70495
η_7	a0[7]	-0.00813	0.00682	-1.19272
η_8	a0[8]	0.03605	0.0074	4.87379
η_9	a0[9]	0.02178	0.00708	3.07583
η_{10}	a0[10]	-0.03769	0.00867	-4.34939
η_{11}	a0[11]	0.01625	0.00912	1.78202
η_{12}	a0[12]	0.05256	0.00915	5.74288

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Nord Pool System Price: Nonlinear Error-Shock Analysis

5. The SNP model and Empirical Findings Non-linear Error-Shock Analysis
5.1 The SNP Density Projection (*f (y | x)*)

The model characteristics with associated standardized residual characteristics:

Number of observations:	7880	s_n	1.0995377
Log Likelihood	-3679.750596	*aic*	1.1034647
		bic	1.1171580
Largest eigenvalue of mean function companion matrix:			0.894485
Largest eigenvalue of variance function P&Q companion m:			0.981629

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Nord Pool System Price: Nonlinear Error-Shock Analysis

5. The SNP model and Empirical Findings Non-linear Error-Shock Analysis
5.1 The SNP Density Projection (*f (y | x)*)

The model characteristics with associated standardized residual characteristics:

Table 4. Residual Statistics for the seasonal adjusted Nordic/Balkan Electricty System Price

	Mean /	Median /	Maximum /	Moment	Quantile	Quantile	Cramer-	Serial dependence	
Residuals	Mode	Std.dev.	Minimum	Kurt/Skew	Kurt/Skew	Normal	von-Mises	Q(12)	$Q^2(12)$
	0.00027	0.02858	11.24376	9.18943	0.04684	0.72127	10.53337	7.8742	8.132
		1.00002	-7.11917	0.10340	0.00082	{0.6972}	{0.0000}	{0.7950}	{0.7755}
	BDS-statistic (ε=1)				ARCH	RESET	Joint	VaR	CVaR
	m=2	m=3	m=4	m=5	(12)	(12;6)	Bias	5%/ 1%	5%/ 1%
	0.85432	2.75108	2.08730	1.78041	10.38351	18.38253	6.55309	-1.5050 %	-2.3628 %
	{0.3929}	{0.0059}	{0.0369}	{0.0750}	{0.5824}	{0.1046}	{0.0876}	-2.7594 %	-3.8689 %

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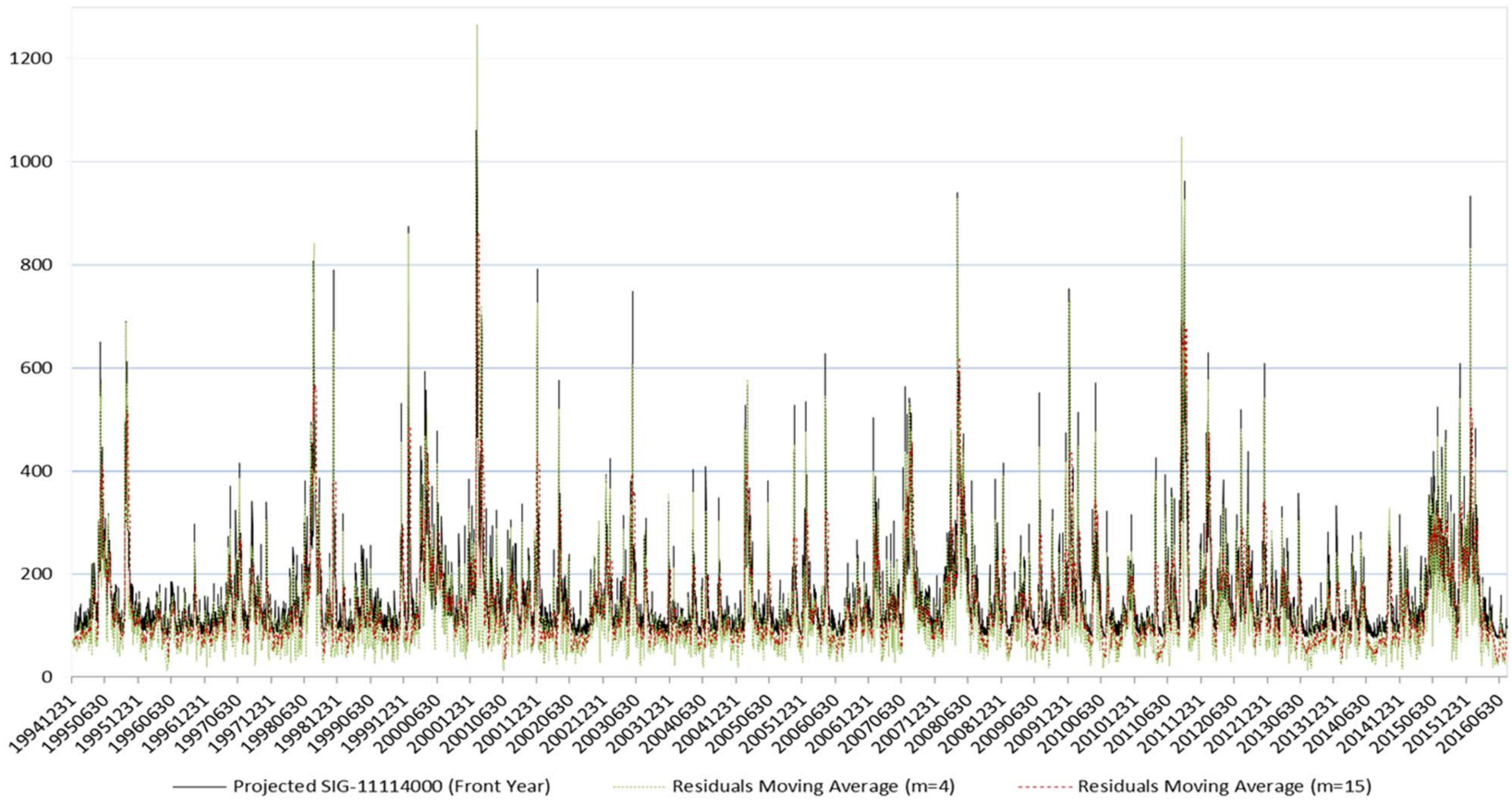
Nord Pool System Price: Nonlinear Error-Shock Analysis

5. The SNP model and Empirical Findings Non-linear Error-Shock Analysis
5.1 The SNP Density Projection (*f (y | x)*). Characteristics.

Model characteristics from (standardized) residuals:

Projected System Price Conditional Volatility and Residual (AR1) Moving Average (*m*=4 and 15)



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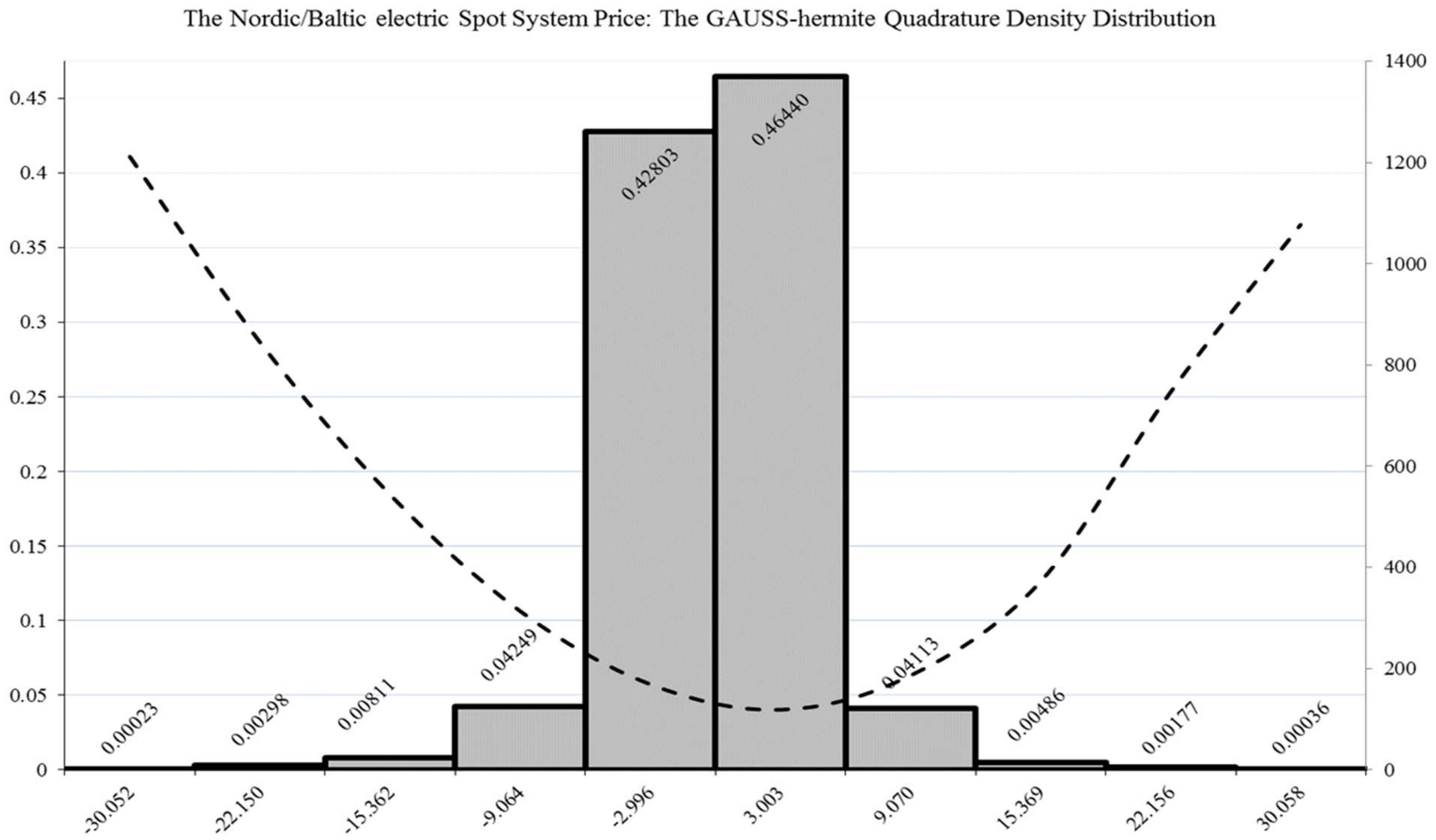

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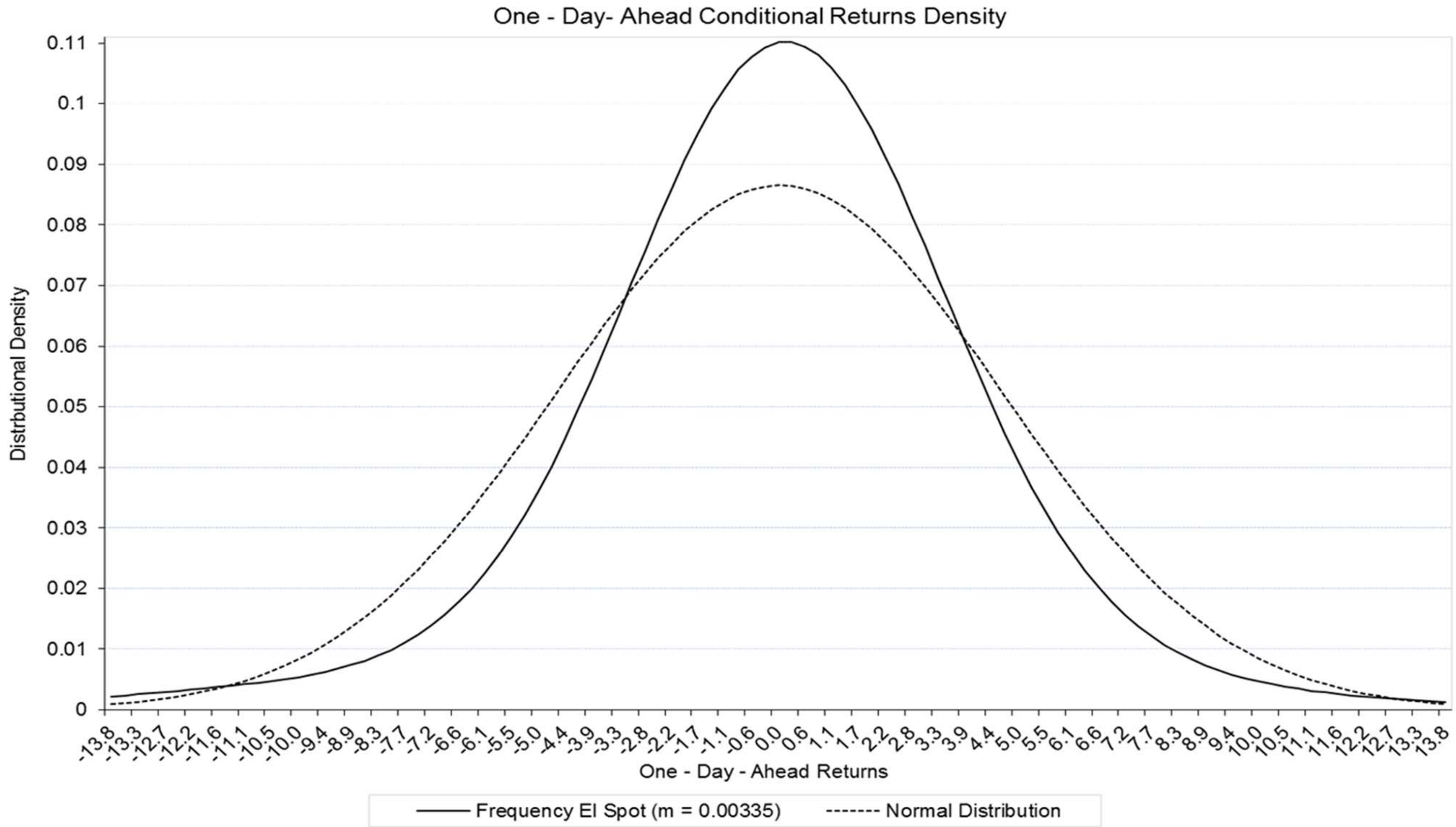
5. The SNP model and Empirical Findings Non-linear Error-Shock Analysis
5.1 The SNP Density Projection (*f (y | x)*). Characteristics.

Model characteristics: one-day-ahead returns (with normal density):



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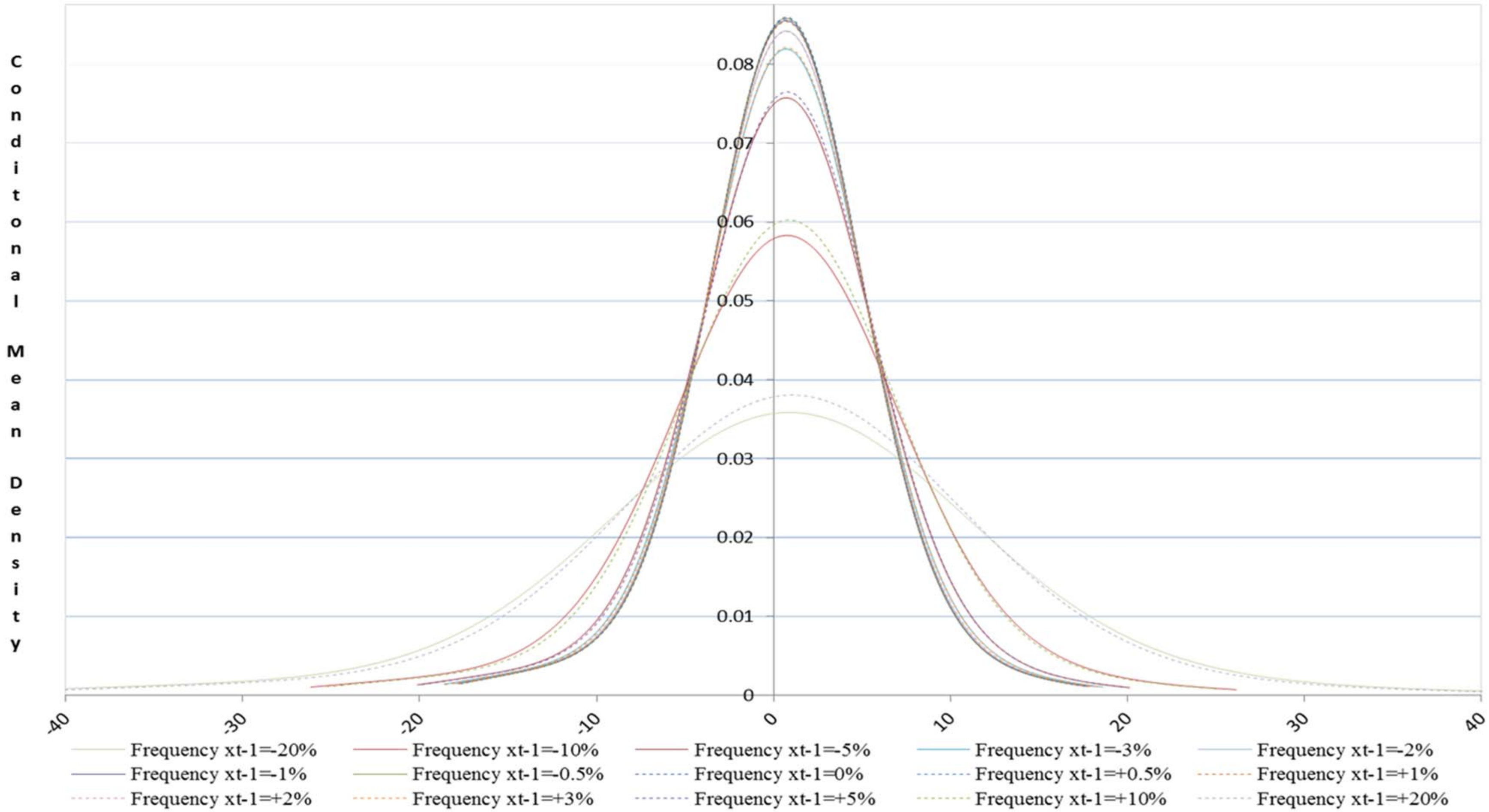
5.1 The SNP Density Projection ($f(y \mid x)$). Characteristics

Model characteristics: one-day-ahead returns conditional on x_{t-1} = -20%,…,20%.

One-step-ahead density $f_K(y_t|x_{t-1},q)$ x_{t-1}= -20,-10,-5,-3,-2,-1,-.5, 0, .5, 1, 2, 3, 5, 10, 20%



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5. The SNP model and Empirical Findings Non-linear Error-Shock Analysis

5.1 The SNP Density Projection ($f\,(y \mid x)$)

The model characteristics comments:

The residuals comments:

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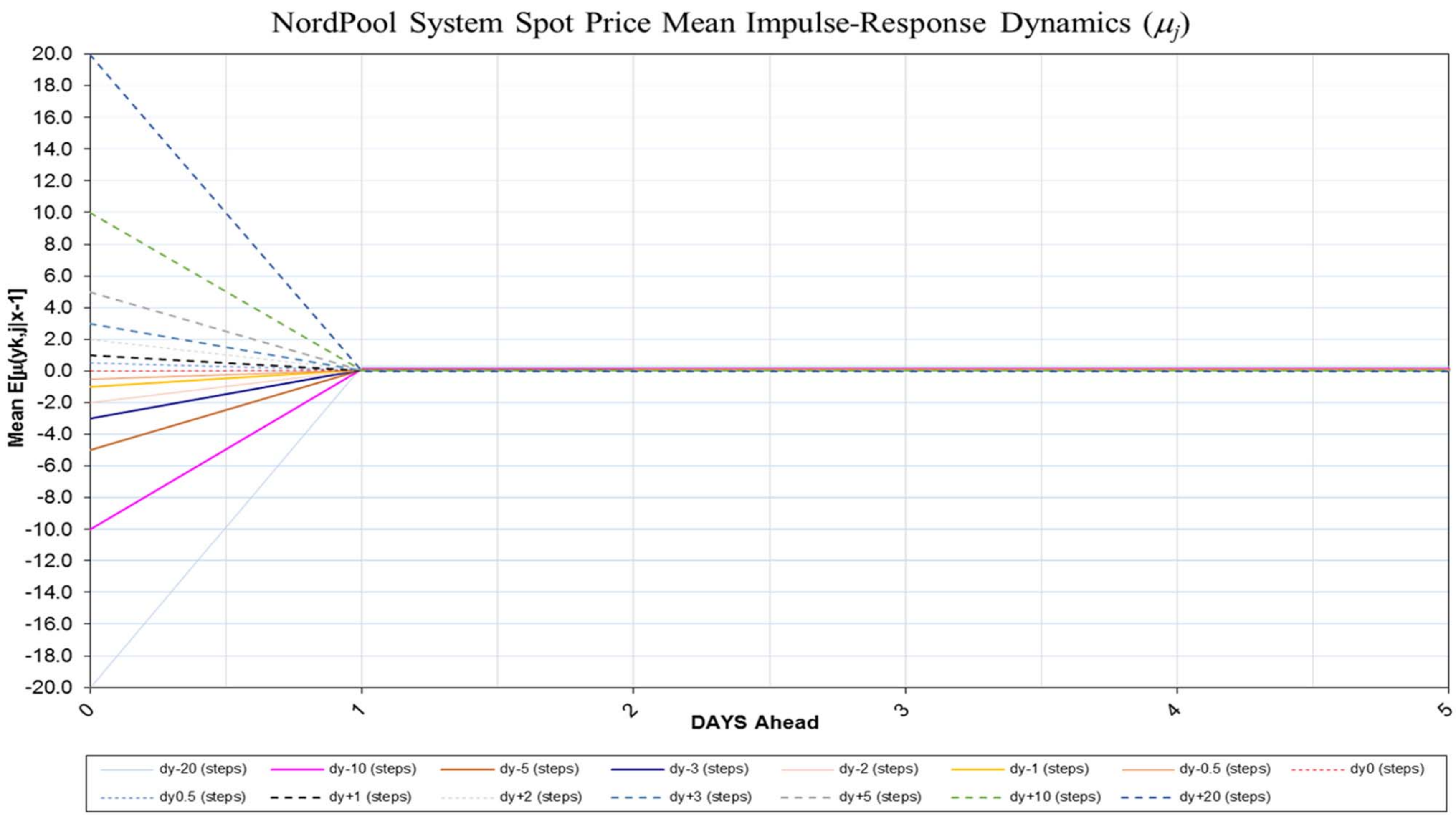
5. Empirical / Findings Non-linear Error-Shock Analysis
5.2 The Impulse-Response Analysis

Mean Impulse-Response Functions: $\left\{ \hat{y}_j^{\,i} - \hat{y}_j^{\,0} \right\}_{j=1}^{5}$ for i = -20%, …, 0, …, 20%



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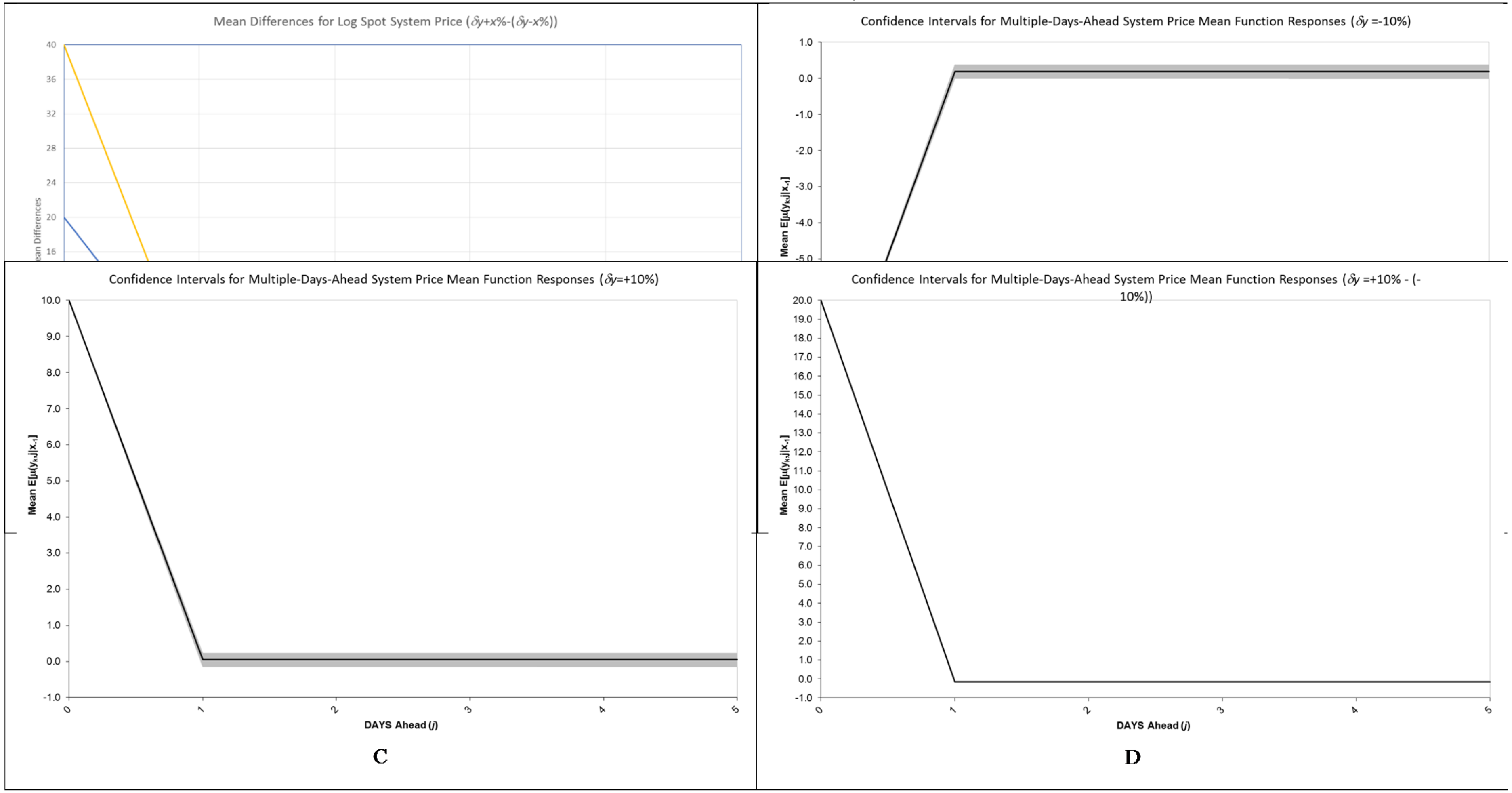
5. Empirical / Findings Non-linear Error-Shock Analysis
5.2 The Impulse-Response Analysis

Mean Impulse Response Functions: $\left\{\hat{y}^{i}_{j} - \hat{y}^{0}_{j}\right\}_{j=1}^{5}$ for i = -20%, …, 0, …, 20%



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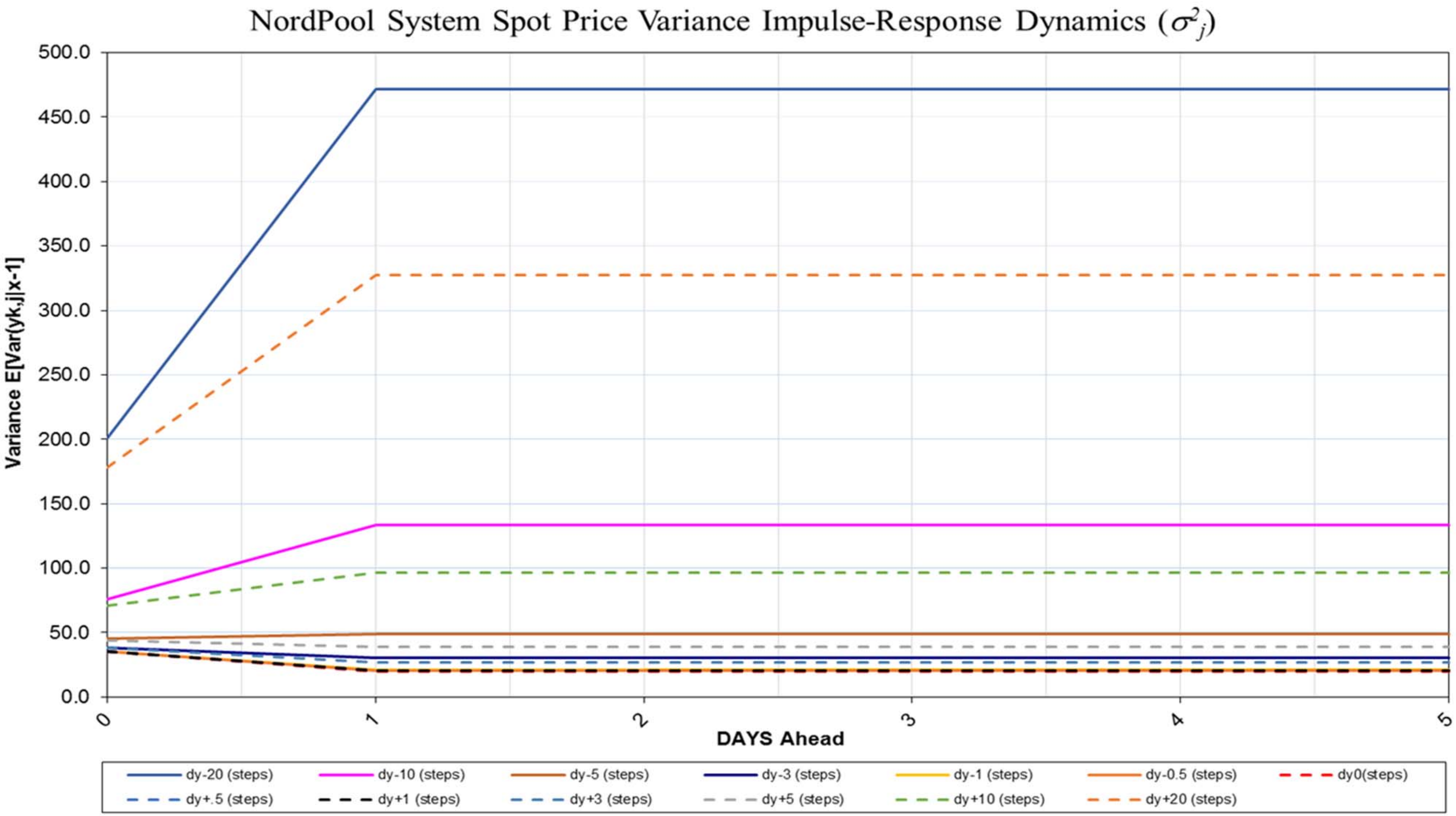
Nord Pool System Price: Nonlinear Error-Shock Analysis

5. Empirical / Findings Non-linear Error-Shock Analysis
5.2 The Impulse-Response Analysis

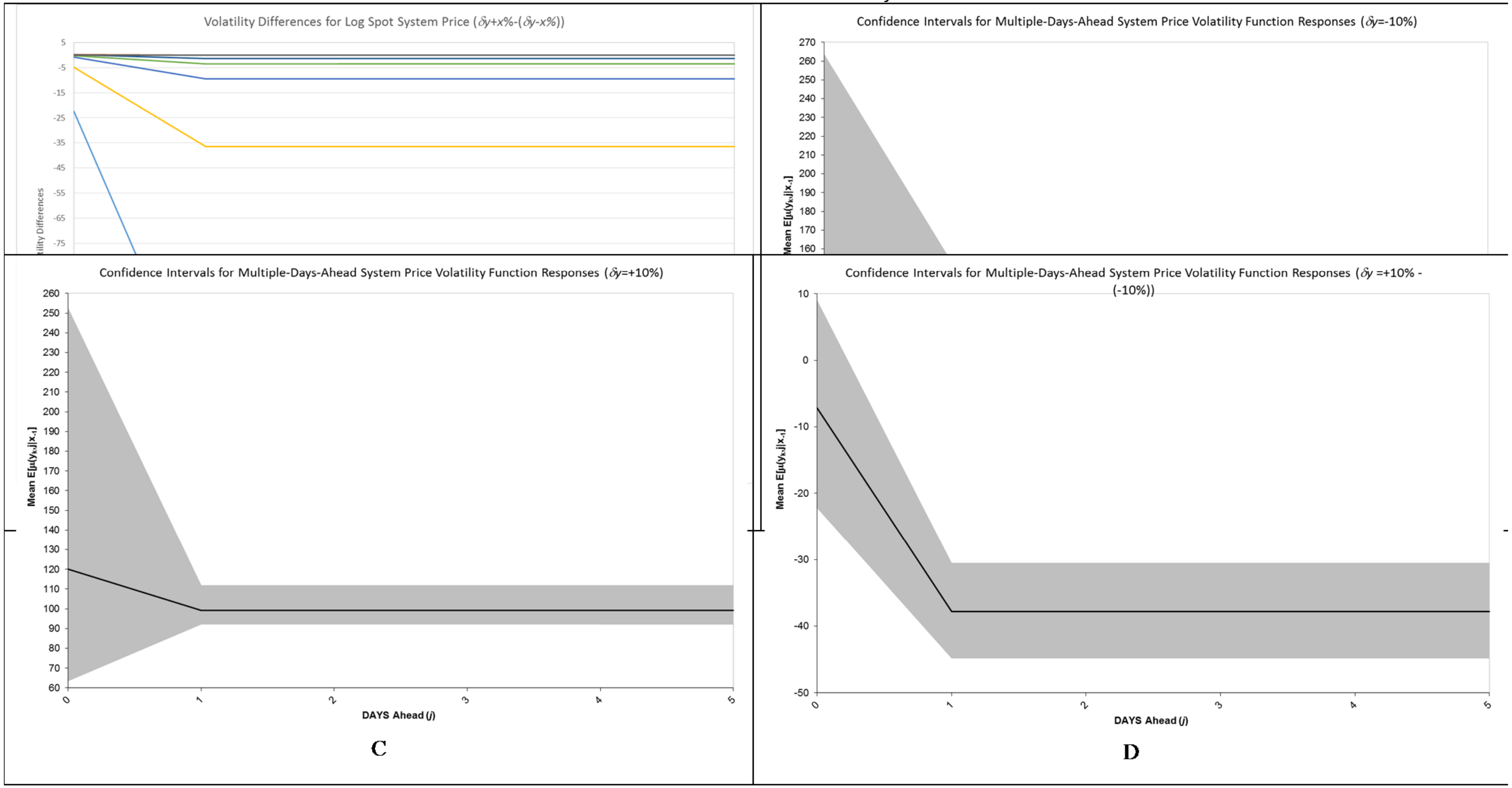
Volatility Impulse Response Functions: $\left\{\hat{\psi}_j^i - \hat{\psi}_j^0\right\}_{j=1}^{5}$ for i = -20%, …, 0, …, 20%



Nord Pool System Price: Nonlinear Error-Shock Analysis

5. Empirical / Findings Impulse-Response Analysis
5.2 The Impulse-Response Analysis

Volatility Impulse Response Functions: $\left\{\psi^{i}_{j}-\psi^{0}_{j}\right\}_{j=1}^{5}$ for i = -20%, …, 0, …, 20%



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Nord Pool System Price: Nonlinear Error-Shock Analysis

5. Empirical / Findings Impulse-Response Analysis
5.2 The Impulse-Response Analysis

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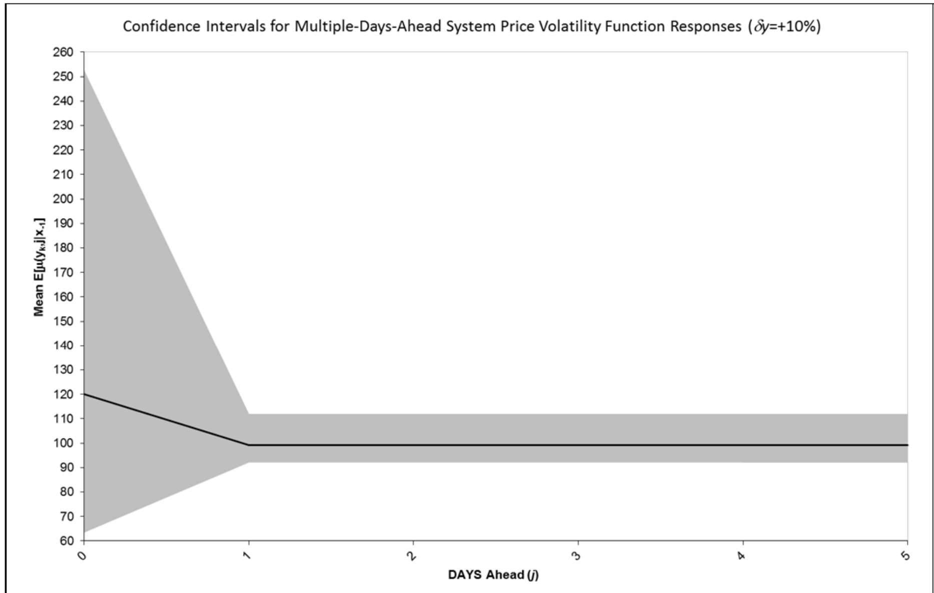

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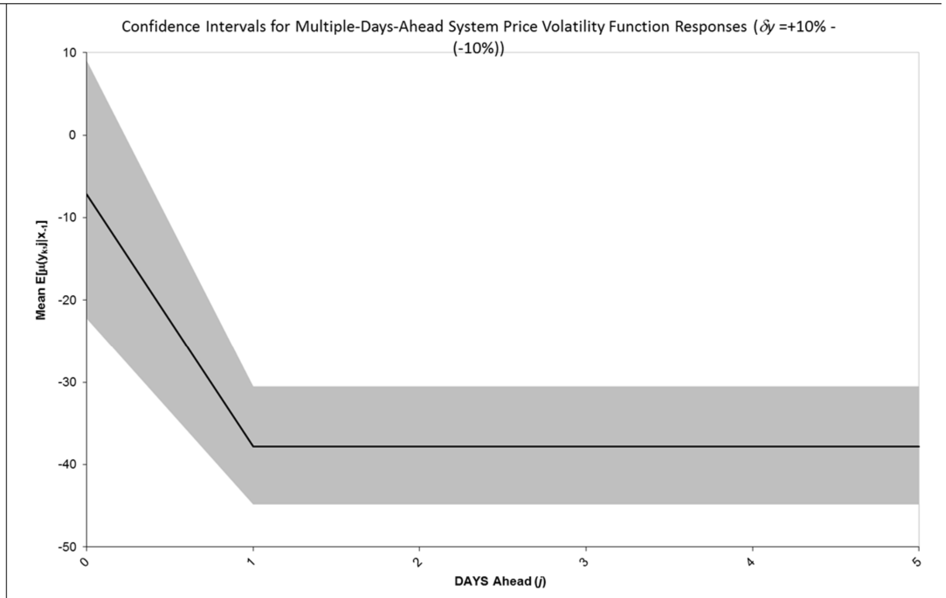

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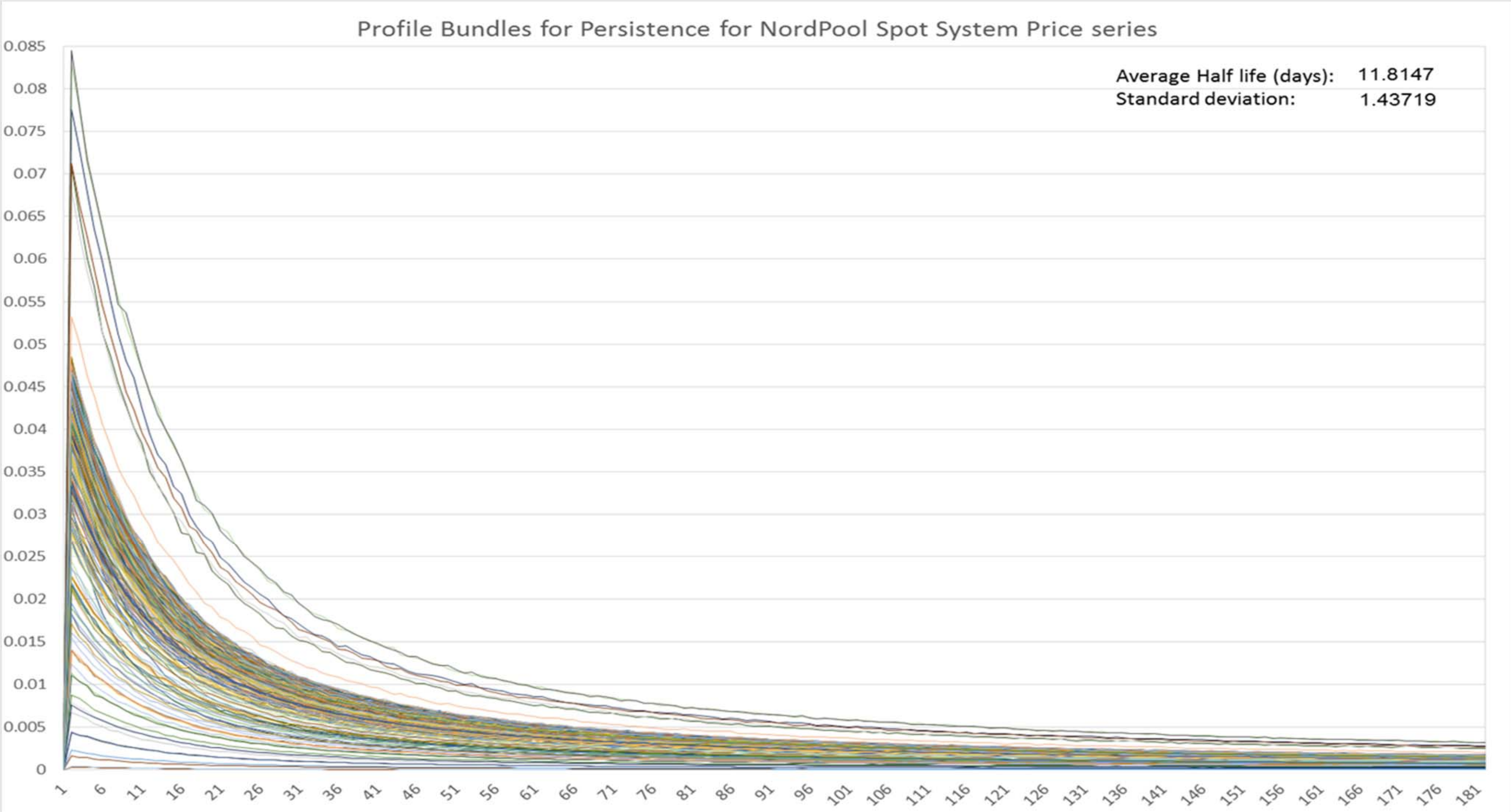
Nord Pool System Price: Nonlinear Error-Shock Analysis

5. Empirical / Findings Impulse-Response Analysis

5.2 Profile bundles for persistence. The half-life definition is from Engle (2001)

$$\tau = k : | h_{t+k|t} - \sigma^2 | = \frac{1}{2} | h_{t+1|t} - \sigma^2 | .$$



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Nord Pool System Price: Nonlinear Error-Shock Analysis

5. Empirical / Findings Impulse-Response Analysis
5.3 The Impulse-Response Analysis for sub-period 2008-2016

Table 3. NordPool Spot Electricity System Price

Statistical Model SNP-14,111,12, 000 -fit; semi-parametric-GARCH model

Mean Equation			Standard	
Var	SNP Coeff.	Mode	error	t-statistics
Mean Correlation				
η_{13}	B(1,1)	-0.00412	0.02029	-0.20324
η_{14}	B(1,2)	-0.12771	0.01802	-7.08924
η_{15}	B(1,3)	-0.05509	0.01806	-3.04998
η_{16}	B(1,4)	-0.03544	0.01732	-2.0456
η_{17}	B(1,5)	-0.01709	0.01690	-1.01141
η_{18}	B(1,6)	0.03301	0.01636	2.01699
η_{19}	B(1,7)	0.11627	0.01622	7.16753
η_{20}	B(1,8)	0.03176	0.01669	1.90276
η_{21}	B(1,9)	0.00992	0.01621	0.61204
η_{22}	B(1,10)	-0.00906	0.01620	-0.55965
η_{23}	B(1,11)	-0.02616	0.01551	-1.68619
η_{24}	B(1,12)	-0.02888	0.01514	-1.9067
η_{25}	B(1,13)	-0.01198	0.01412	-0.84827
η_{26}	B(1,14)	0.06762	0.01336	5.05969

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Nord Pool System Price: Nonlinear Error-Shock Analysis

5. Empirical / Findings Impulse-Response Analysis

5.3 The Impulse-Response Analysis for sub-period 2008-2016

Table 3. NordPool Spot Electricity System Price

Statistical Model SNP-14,111,12, 000 -fit; semi-parametric-GARCH model

Mean Equation			Standard	
Var	SNP Coeff.	Mode	error	t-statistics
Variance Equation				
η_{27}	R0[1]	0.19355	0.02392	8.09323
η_{28}	P[1,1]	0.49276	0.06581	7.48745
η_{29}	Q[1,1]	0.90662	0.00879	103.18305
η_{30}	V[1,1]	-0.39408	0.07566	-5.20831
η_{31}	W[1,1]	0.79339	0.09259	8.56934

Number of observations:	3132	s_n	1.0769270
Log Likelihood	-3388.012202	*aic*	1.0867807
		bic	1.1166076
Largest eigenvalue of mean function companion matrix:			0.887869
Largest eigenvalue of variance function P&Q companion m:			1.064760

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Nord Pool System Price: Nonlinear Error-Shock Analysis

5. Empirical / Findings Impulse-Response Analysis

5.3 The Impulse-Response Analysis for sub-period 2008-2016

Table 3. NordPool Spot Electricity System Price

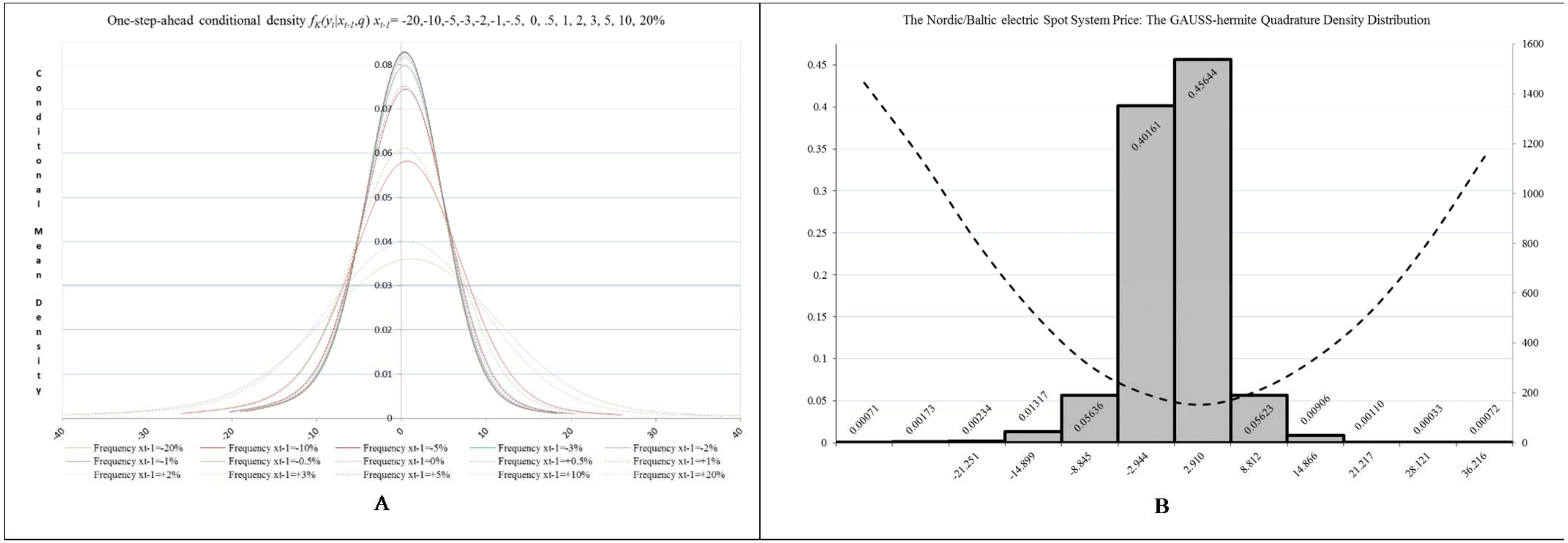
Statistical Model SNP-14,111,12, 000 -fit; semi-parametric-GARCH model

Mean Equation Var	SNP Coeff.	Mode	Standard error	t-statistics
Hermite Polynoms				
η_1	$a_0[1]$	-0.00501	0.01122	-0.44677
η_2	$a_0[2]$	-0.24748	0.01891	-13.08827
η_3	$a_0[3]$	-0.02960	0.04264	-0.69413
η_4	$a_0[4]$	0.18462	0.0768	2.40405
η_5	$a_0[5]$	-0.00179	0.07622	-0.02351
η_6	$a_0[6]$	-0.11030	0.04949	-2.22874
η_7	$a_0[7]$	-0.02973	0.0503	-0.59104
η_8	$a_0[8]$	0.05963	0.012	4.96755
η_9	$a_0[9]$	0.02792	0.04193	0.66589
η_{10}	$a_0[10]$	-0.06803	0.05928	-1.14754
η_{11}	$a_0[11]$	0.00593	0.07072	0.08384
η_{12}	$a_0[12]$	0.05315	0.03331	1.59541

Nord Pool System Price: Nonlinear Error-Shock Analysis

5. Empirical / Findings Impulse-Response Analysis

5.3 The Impulse-Response Analysis for sub-period 2008-2016



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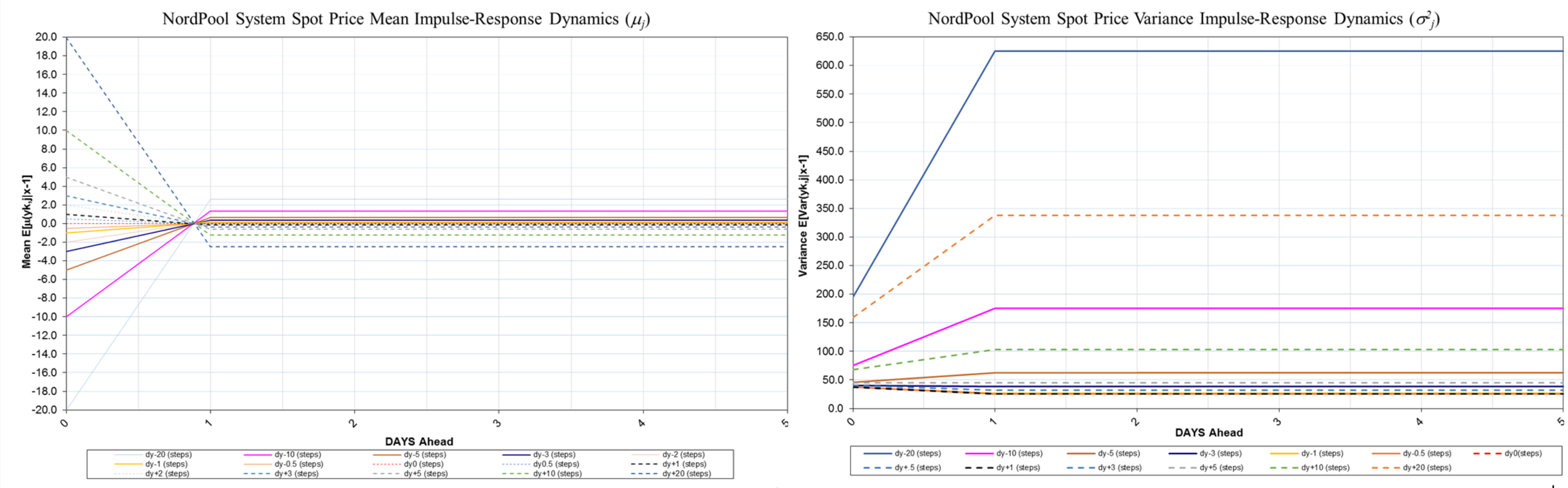
Nord Pool System Price: Nonlinear Error-Shock Analysis

5. Empirical / Findings Impulse-Response Analysis
5.3 The Impulse-Response Analysis for sub-period 2008-2016



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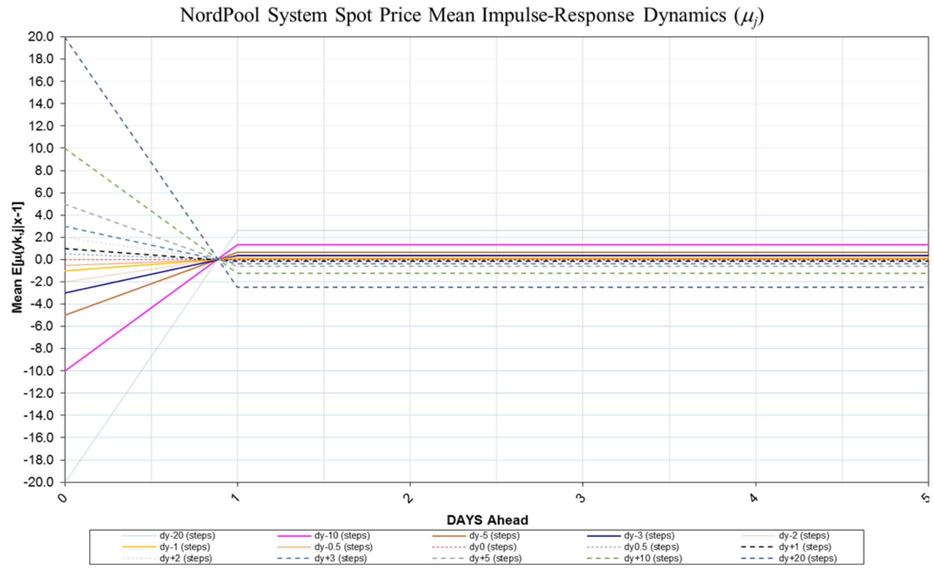
Nord Pool System Price: Nonlinear Error-Shock Analysis

5. Empirical / Findings Impulse-Response Analysis
5.3 The Impulse-Response Analysis for sub-period 2008-2016



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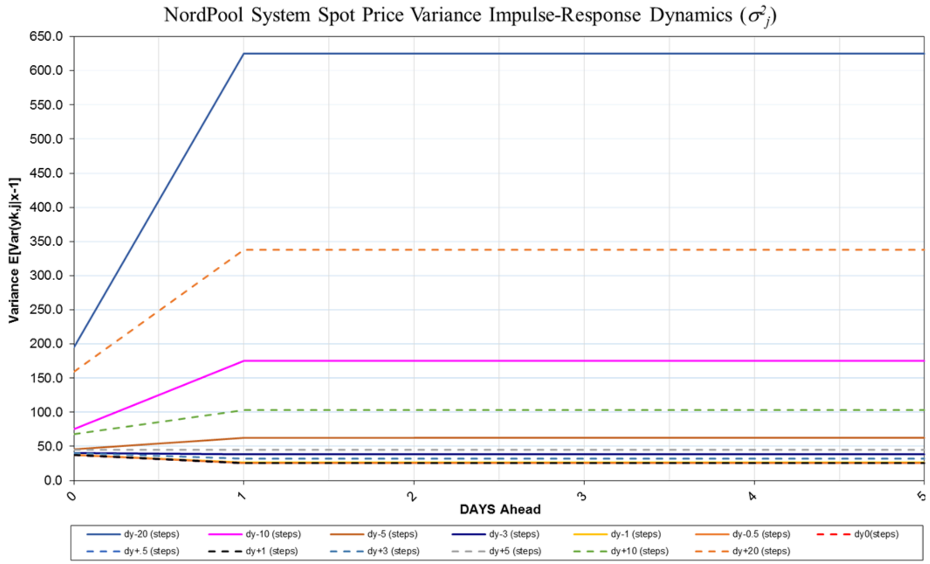

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Nord Pool System Price: Nonlinear Error-Shock Analysis

5. Empirical / Findings Impulse-Response Analysis

Table 5. Impulse-Response Functions (Volatility) Nordic/Baltic Spot System Price

Panel A. Volatility Period 1994 - 2016

Days / Δ	δy-20	δy-10	δy-5	δy-3	δy-1	δy-.5	δy+.5	δy+1	δy+3	δy+5	δy+10	δy+20
0	200.78175	75.98636	45.02147	38.49397	35.31187	35.06853	35.11649	35.40743	38.35094	44.14202	71.07786	178.34163
1	471.32747	133.21172	48.49447	30.35001	20.86496	20.22343	20.17102	20.72406	26.79556	39.04195	96.67309	327.71467
2	471.33605	133.21332	48.49443	30.34966	20.86451	20.22298	20.17059	20.72366	26.79532	39.04200	96.67437	327.72058
3	471.34324	133.21446	48.49427	30.34929	20.86414	20.22262	20.17029	20.72339	26.79526	39.04225	96.67588	327.72665
4	471.35085	133.21601	48.49437	30.34910	20.86383	20.22232	20.17000	20.72312	26.79512	39.04234	96.67697	327.73160
5	471.35977	133.21845	48.49499	30.34927	20.86367	20.22210	20.16973	20.72284	26.79484	39.04215	96.67741	327.73498

Panel B. Volatility Sub-Period 2008 - 2016

Days / Δ	δy-20	δy-10	δy-5	δy-3	δy-1	δy-.5	δy+.5	δy+1	δy+3	δy+5	δy+10	δy+20
0	195.62030	75.41405	46.05012	40.03059	37.30215	37.14053	37.26580	37.55268	40.19514	45.22951	68.28006	159.22949
1	625.22229	174.92591	62.47763	38.53563	26.06388	25.34653	25.37055	25.96885	32.26359	44.80082	103.45476	337.80921
2	625.23600	174.92884	62.47798	38.53547	26.06350	25.34616	25.37019	25.96851	32.26343	44.80096	103.45623	337.81582
3	625.24864	174.93144	62.47824	38.53530	26.06320	25.34586	25.36993	25.96827	32.26338	44.80123	103.45785	337.82251
4	625.26006	174.93374	62.47847	38.53516	26.06295	25.34562	25.36972	25.96810	32.26339	44.80154	103.45942	337.82879
5	625.27473	174.93792	62.47969	38.53563	26.06292	25.34548	25.36950	25.96786	32.26315	44.80140	103.46002	337.83288

Panel C. Volatility differences Period 1994 - 2016 vrs Sub-Period 2008 - 2016

Days / Δ	δy-20	δy-10	δy-5	δy-3	δy-1	δy-.5	δy+.5	δy+1	δy+3	δy+5	δy+10	δy+20
0	5.16144	0.57232	-1.02866	-1.53662	-1.99028	-2.07200	-2.14931	-2.14525	-1.84420	-1.08750	2.79781	19.11214
1	-153.89482	-41.71419	-13.98316	-8.18562	-5.19891	-5.12311	-5.19954	-5.24479	-5.46804	-5.75887	-6.78167	-10.09454
2	-153.89995	-41.71552	-13.98354	-8.18581	-5.19899	-5.12318	-5.19960	-5.24485	-5.46810	-5.75896	-6.78186	-10.09524
3	-153.90541	-41.71698	-13.98397	-8.18602	-5.19906	-5.12324	-5.19964	-5.24488	-5.46812	-5.75898	-6.78197	-10.09586
4	-153.90921	-41.71773	-13.98409	-8.18606	-5.19912	-5.12330	-5.19972	-5.24498	-5.46827	-5.75920	-6.78245	-10.09719
5	-153.91496	-41.71947	-13.98470	-8.18637	-5.19924	-5.12338	-5.19977	-5.24502	-5.46831	-5.75925	-6.78261	-10.09790

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Nord Pool System Price: Nonlinear Error-Shock Analysis

5. Empirical / Findings Impulse-Response Analysis

Table 6 Volatility and Volatilty differences Sup-Norm ε -bands (95%)

	Volatility	ε - bands			Volatility	ε - bands			Volatility	ε - bands		
	δy-10	95%	5%	50%	δy+10	0.95	0.05	0.50	$\delta y(10\text{-}(-10)$	95%	5%	50%
0	75.99	263.57	69.85	92.57	71.08	252.41	63.55	85.64	-4.908501	8.99	-22.19	-6.734
1	133.21	148.29	129.11	136.22	96.67	111.85	92.22	97.87	-36.53863	-30.53	-44.79	-37.64
2	133.21	148.30	129.11	136.22	96.67	111.85	92.22	97.87	-36.53895	-30.53	-44.79	-37.64
3	133.21	148.30	129.11	136.22	96.68	111.85	92.22	97.87	-36.53858	-30.53	-44.78	-37.64
4	133.22	148.30	129.11	136.22	96.68	111.85	92.22	97.87	-36.53904	-30.53	-44.78	-37.64
5	133.22	148.30	129.12	136.22	96.68	111.85	92.22	97.87	-36.54104	-30.53	-44.79	-37.64

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Nord Pool System Price: Nonlinear Error-Shock Analysis

5. Empirical / Findings Impulse-Response Analysis

Table 7. Volatility Asymmetry Nordic/Baltic Spot System Price

Panel A. Asymmetric Differences for the Period 1994 - 2016

Days/Δ	δy (+20-(-20))	δy (+10-(-10))	δy (+5-(-5))	δy (+3-(-3))	δy (+2-(-2))	δy (+1-(-1))	δy (+0.5-(-0.5))
0	-22.440119	-4.908501	-0.879449	-0.143033	0.108672	0.095561	0.047963
1	-143.612794	-36.538629	-9.452522	-3.554450	-1.358719	-0.140900	-0.052411
2	-143.615471	-36.538954	-9.452433	-3.554335	-1.358611	-0.140846	-0.052385
3	-143.616588	-36.538579	-9.452020	-3.554027	-1.358398	-0.140745	-0.052335
4	-143.619248	-36.539043	-9.452037	-3.553987	-1.358342	-0.140714	-0.052320
5	-143.624788	-36.541037	-9.452838	-3.554423	-1.358624	-0.140832	-0.052370

Panel B. Asymmetric Differences for the Sub-Period 2008 - 2016

Days/Δ	δy (+20-(-20))	δy (+10-(-10))	δy (+5-(-5))	δy (+3-(-3))	δy (+2-(-2))	δy (+1-(-1))	δy (+0.5-(-0.5))
0	-36.390812	-7.133989	-0.820612	0.164548	0.426244	0.250530	0.125274
1	-287.413075	-71.471152	-17.676808	-6.272038	-2.641619	-0.095027	0.024017
2	-287.420176	-71.472612	-17.677018	-6.272041	-2.641568	-0.094987	0.024036
3	-287.426137	-71.473591	-17.677014	-6.271925	-2.641441	-0.094923	0.024068
4	-287.431269	-71.474319	-17.676927	-6.271770	-2.641296	-0.094852	0.024103
5	-287.441847	-71.477898	-17.678288	-6.272486	-2.641753	-0.095055	0.024024

Panel C. Asymmetric Differences between Period 1994 - 2016 and Sub-period 2008 - 2016

Days/Δ	δy (+20-(-20))	δy (+10-(-10))	δy (+5-(-5))	δy (+3-(-3))	δy (+2-(-2))	δy (+1-(-1))	δy (+0.5-(-0.5))
0	13.9506929	2.2254888	-0.0588378	-0.3075812	-0.3175723	-0.1549689	-0.0773117
1	143.8002810	34.9325232	8.2242861	2.7175880	1.2829000	-0.0458728	-0.0764279
2	143.8047049	34.9336576	8.2245847	2.7177059	1.2829575	-0.0458589	-0.0764215
3	143.8095491	34.9350121	8.2249933	2.7178975	1.2830435	-0.0458225	-0.0764038
4	143.8120216	34.9352752	8.2248898	2.7177833	1.2829543	-0.0458617	-0.0764238
5	143.8170591	34.9368608	8.2254504	2.7180624	1.2831293	-0.0457770	-0.0763938

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Nord Pool System Price: Nonlinear Error-Shock Analysis

5. Empirical / Findings Impulse-Response Analysis

Table 7. Volatility Asymmetry Nordic/Baltic Spot System Price

Panel A. Asymmetric Differences for the Period 1994 - 2016

Days/Δ	δy (+20-(-20))	δy (+10-(-10))	δy (+5-(-5))	δy (+3-(-3))	δy (+2-(-2))	δy (+1-(-1))	δy (+0.5-(-0.5))
0	-22.440119	-4.908501	-0.879449	-0.143033	0.108672	0.095561	0.047963
1	-143.612794	-36.538629	-9.452522	-3.554450	-1.358719	-0.140900	-0.052411
2	-143.615471	-36.538954	-9.452433	-3.554335	-1.358611	-0.140846	-0.052385
3	-143.616588	-36.538579	-9.452020	-3.554027	-1.358398	-0.140745	-0.052335
4	-143.619248	-36.539043	-9.452037	-3.553987	-1.358342	-0.140714	-0.052320
5	-143.624788	-36.541037	-9.452838	-3.554423	-1.358624	-0.140832	-0.052370

Panel B. Asymmetric Differences for the Sub-Period 2008 - 2016

Days/Δ	δy (+20-(-20))	δy (+10-(-10))	δy (+5-(-5))	δy (+3-(-3))	δy (+2-(-2))	δy (+1-(-1))	δy (+0.5-(-0.5))
0	-36.390812	-7.133989	-0.820612	0.164548	0.426244	0.250530	0.125274
1	-287.413075	-71.471152	-17.676808	-6.272038	-2.641619	-0.095027	0.024017
2	-287.420176	-71.472612	-17.677018	-6.272041	-2.641568	-0.094987	0.024036
3	-287.426137	-71.473591	-17.677014	-6.271925	-2.641441	-0.094923	0.024068
4	-287.431269	-71.474319	-17.676927	-6.271770	-2.641296	-0.094852	0.024103
5	-287.441847	-71.477898	-17.678288	-6.272486	-2.641753	-0.095055	0.024024

Panel C. Asymmetric Differences between Period 1994 - 2016 and Sub-period 2008 - 2016

Days/Δ	δy (+20-(-20))	δy (+10-(-10))	δy (+5-(-5))	δy (+3-(-3))	δy (+2-(-2))	δy (+1-(-1))	δy (+0.5-(-0.5))
0	13.9506929	2.2254888	-0.0588378	-0.3075812	-0.3175723	-0.1549689	-0.0773117
1	143.8002810	34.9325232	8.2242861	2.7175880	1.2829000	-0.0458728	-0.0764279
2	143.8047049	34.9336576	8.2245847	2.7177059	1.2829575	-0.0458589	-0.0764215
3	143.8095491	34.9350121	8.2249933	2.7178975	1.2830435	-0.0458225	-0.0764038
4	143.8120216	34.9352752	8.2248898	2.7177833	1.2829543	-0.0458617	-0.0764238
5	143.8170591	34.9368608	8.2254504	2.7180624	1.2831293	-0.0457770	-0.0763938

Nord Pool System Price: Nonlinear Error-Shock Analysis

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Nord Pool System Price: Nonlinear Error-Shock Analysis

6. Summary and Findings

We have modelled a conditional mean and volatility model for the adjusted and stationary System Price from the Nord Pool Electricity market.

The optimal SNP specification becomes: 14,1,1,1,12,0,0,0. The battery of specification test for the residuals suggest an appropriate description of the adjusted time series.

Results for the specification:

- drift is close to zero (positive)
- serial correlation for the mean up to 14 days / lags
- mean reversion
- conditional heteroscedasticity

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Nord Pool System Price: Nonlinear Error-Shock Analysis

6. Summary Findings

We have modelled a conditional model for the adjusted and stationary System Price from the Nord Pool Electricity market.

Results for the nonlinear error-shock analysis and the conditional mean:

- immediate dissipation which suggest linearity in the conditional mean equation

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Nord Pool System Price: Nonlinear Error-Shock Analysis

6. Summary Findings

We have modelled a conditional model for the adjusted and stationary System Price from the Nord Pool Electricity market.

Results for the non-linear error-shock analysis and volatility:

- quite high conditional volatility responses from large price changes (shocks).
- for smaller changes in the price (shocks), the volatility show modest increases and even decreases relative to the longer period, inducing quite small responses
- asymmetry is low for small price changes (>-5% and <5%) but become quite severe when price changes move to > 10% and < -10%.
- persistence of shocks are short (approx. 12 days).

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Nord Pool System Price: Nonlinear Error-Shock Analysis

6. Summary Findings

We have modelled a conditional mean and volatility model for the adjusted and stationary System Price from the Nord Pool Electricity market.

Results for the impulse-response analysis and the sub-period 2008 - 2016:

- immediate dissipation suggesting linearity in the conditional mean equation
- stronger negative correlation quite symmetric around zero
- for small price changes the volatility and asymmetry has decreased and in some instances turned positive (maturity)
- for larger price changes the volatility and asymmetry has increased (more severe effects from shocks). The asymmetry has become more negative