Determinants of Volatility Smile: the Case of Crude Oil Options

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- Important to understand how these markets work.
- Surprisingly, there is not much previous research on crude oil options.

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- Implied volatility: volatility of crude oil future which, when input in an option pricing model (such as Black–Scholes) will return a theoretical value equal to the current market price of the option.

Introduction

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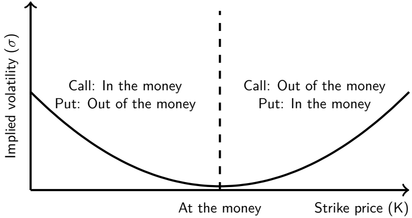
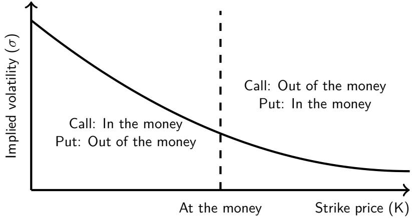
- But option contracts also differ with respect to strike price.
- Previous research has shown that many markets exhibit ‘volatility smiles’
    - Implied volatility differs across strike prices.

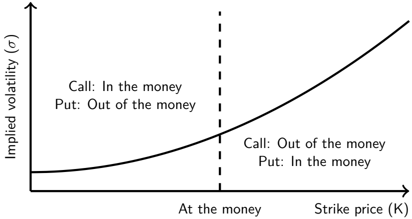
Relationships between implied volatility and moneyness



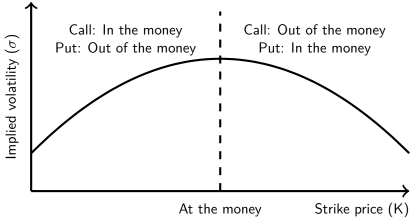
(a) **Smile**



(b) **Reverse skew (Smirk)**



(c) **Forward skew**



(d) **Frown**

This Study

- We construct a data set to study volatility smiles for WTI crude oil.
- Our first-stage regression estimates a quadratic approximation of implied volatility as a function of moneyness.
  - The coefficient on the quadratic term should be positive if the market exhibits volatility smiles.
- Our second-stage regression investigates correlations between the estimated parameters and a list of explanatory variables.

Preview of Results

- We find evidence of volatility smiles in the crude oil market.
  - This is especially true for contracts with a long maturity (4-12 months).
- We find that volatility smiles tend to happen during times of high basis and high hedging pressure of the underlying futures contract.
  - This is contrasted to the net buying pressure explanation of Bollen and Whaley (2004) and the transaction cost explanation discussed in Longstaff (1995), Dumas et al. (1998) and Pena et al. (1999).
  - We show that futures return distribution tends to exhibit fatter tails during times of high basis and high hedging pressure.
- We find that also transaction costs play a role.

Related Literature 1/2

- Alternatives to the Black-Scholes model:
  - Implied binomial tree framework
  - Constant elasticity of variance model
    Cox and Ross (1976), Emanuel and MacBeth (1982)
  - Stochastic volatility models
    Hull and White (1987),
  - Jump diffusion models
    Das and Foresi (1996),

Related Literature 2/2

- Explanations for volatility smiles
  - Net buying pressure
    Bollen and Whaley (2004)
  - Limits to arbitrage
    Shleifer and Vishny (1997), Liu and Longstaff (2003), Acharya et al. (2013)
  - Transaction costs and low liquidity
    Longstaff (1995), Dumas et al. (1998), Pena et al. (1999)
- Literature on crude oil futures (underlying asset):
  - Alquist and Kilian (2010), Liu and Tang (2011)
  - Theory of hedging pressure, theory of storage.

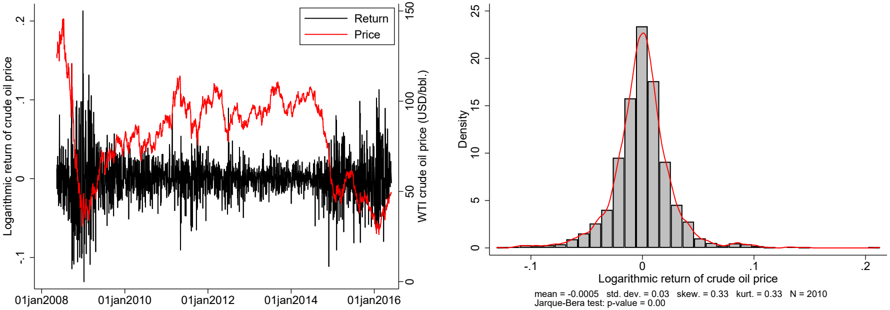
Data

- Daily data on prices of crude oil calls
  - May 13, 2008 - May 31, 2016
  - New York Merchantile Exchange, data from Commodity Research Bureau
  - Maturities: 1-12 months
- Corresponding futures price data. Also trading volumes, holdings of commercial vs non-commercial hedgers, etc.
- Spot price from U.S. Energy Information Administration
- All prices are for West Texas Intermediate (WTI) crude oil.

Section 1:
Descriptive Statistics

Descriptive statistics

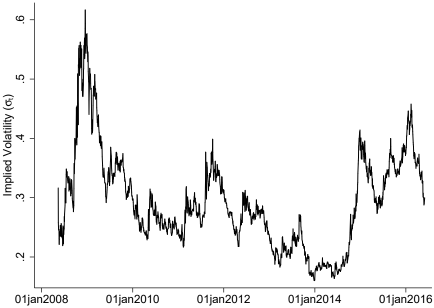
Figure: **WTI crude oil spot price (13.05.2008 - 31.05.2016)**



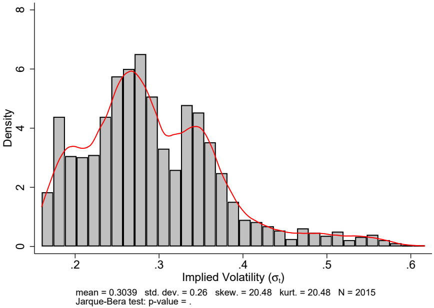
(a) **Price time-series development** (b) **Distribution**

Subfigure (a) shows temporal development of WTI crude oil spot price and logarithmic between 13.05.2008 and 31.05.2016. Subfigure (b) shows the distribution of the logarithmic returns of crude oil price approximated through a histogram and an Epanechnikov kernel density plot.

Figure: **Implied volatility (13.05.2008 - 31.05.2016)**



(a) **Price time-series development**



(b) **Distribution**

Descriptive statistics: List of explanatory variables

Variable	Description
Spread	Mean bid-ask spread for given date and maturity.
OptionVolume	Total trading volume of options for given date and maturity.
Basis	Percentage basis for the underlying crude oil future.
HP	Index of hedging pressure for the underlying crude oil future.
FutureVolume	Total trading volume of the underlying future for a given day and maturity.
Monday	Dummy variable for mondays
DTM	Days to maturity.

Variables

- Basis:

$$\frac{S(t) - F(t)}{F(t)}$$

- $$HP = \frac{\text{Net Short Commercial Hedgers}}{\text{Number of Outstanding Contracts}}$$

Table: **Summary statistics**

Variable	Mean	Std. dev.	Skew.	Kurt.	Min	Median	Max	N
Spread	0.04	0.06	4.49	40.41	0.00	0.01	1.75	305212
OptVol	300.85	772.63	6.91	92.84	1	38	28788	305212
Basis	-0.02	0.05	-1.88	10.31	-0.42	-0.01	0.16	305212
HP	-0.14	0.06	0.27	2.90	-0.29	-0.14	0.04	305212

Summary statistics of Abdi and Ranaldo (2017) spread estimator, option trading volume, basis and hedging pressure conditional on trading volume beging positive.

Section 2:
Methodology and Main Results

First-Stage Regression

We estimate IV as a function of 'moneyness':

$$\sigma_t = \beta_0 + \beta_1 \left(\frac{F}{K}\right) + \beta_2 \left(\frac{F}{K}\right)^2 + \varepsilon_t. \tag{1}$$

This is estimated separately for all maturities and valuation dates, as long as $N \geq 3$.

Examples from the Data

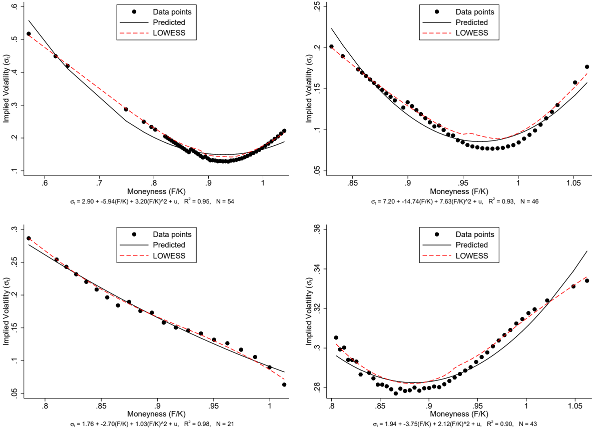


Figure: Top-left: Oct 2, 2014 Top-right: July 7, 2014 Bottom left: April 7, 2016 Bottom right: August 9, 2011

First-Stage Regression Results

Month	$\bar{\beta}_0(\sigma_{\beta_0})$	$\bar{\beta}_1(\sigma_{\beta_1})$	$\bar{\beta}_2(\sigma_{\beta_2})$	$\bar{R}^2$	$\bar{N}(\sigma_N)$	Sub-sample
1	1.22(10.27)	-1.94(20.28)	0.84(10.02)	0.95	36.49(14.57)	906
2	1.10(2.60)	-1.72(5.17)	0.83(2.58)	0.93	47.07(16.34)	1891
3	1.05(0.75)	-1.68(1.69)	0.88(0.90)	0.92	37.97(16.52)	1852
4	1.01(0.70)	-1.63(1.55)	0.90(0.82)	0.95	24.37(13.33)	1780
5	0.92(0.63)	-1.46(1.40)	0.83(0.74)	0.97	14.38(9.82)	1690
6	0.83(0.74)	-1.29(1.63)	0.75(0.86)	0.98	9.63(6.87)	1454
7	0.76(0.52)	-1.13(1.26)	0.67(0.72)	0.98	8.35(6.41)	1068
8	0.69(0.47)	-1.02(1.09)	0.61(0.59)	0.98	7.47(5.57)	733
9	0.66(0.36)	-0.95(0.86)	0.58(0.48)	0.98	7.57(5.08)	527
10	0.63(0.35)	-0.91(0.84)	0.56(0.46)	0.98	8.00(5.39)	414
11	0.60(0.53)	-0.85(1.13)	0.52(0.59)	0.97	7.44(4.86)	349
12	0.58(0.33)	-0.76(0.76)	0.48(0.41)	0.99	6.77(4.43)	301

Second-Stage Regression

$$\beta_{it} = \delta_0 + \sum_{j=1}^{N} \delta_j x_{ijt} + \upsilon_{it} \quad \text{where } i \in \{0, 1, 2\} \tag{2}$$

Explanatory variables:

- Spread
- OptionVolume
- Basis
- HP (hedging pressure)
- FutureVolume
- Monday (dummy)
- DTM

Remarks

- We consider the coefficient on the quadratic term (β_2) to be the main object of interest.
- Contracts with a short maturity behave differently. Here we report pooled results for maturities 4-12 months.
- More detailed results are in the paper.

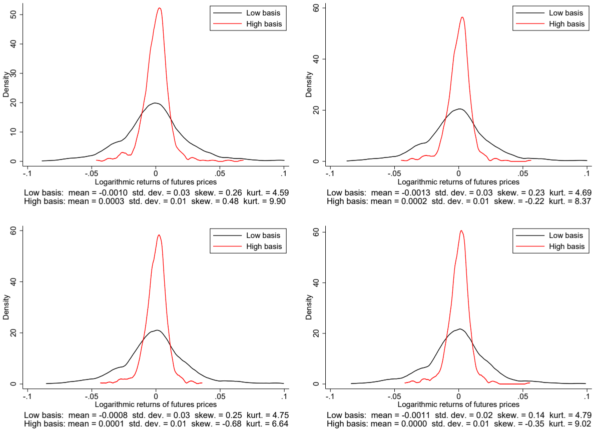
Second-Stage Regression Results

	Dependent Variable			
	β_0		β_2	
Constant	0,84	***	0,59	***
Spread	1,79	***	2,43	***
Option vol	0,01		0,00	
Basis	1,08	***	2,56	***
HP	2,45	***	3,34	***
Future vol	0,01	*	0,03	***
Monday dummy	0,00		0,00	
DTM	0,00	***	0,00	***
Obs	8316		8316	
R-squared	0,18		0,28	

Discussion

- The key finding is that volatility smiles tend to happen during times of high basis or high hedging pressure.
- Transaction costs are also statistically significant.
- Volatility smiles happen more often when DTM is high.

Futures returns across two levels of basis



Section 3:
Robustness Checks

Robustness check 1

- We classify daily IV patterns into 4 groups :
  - Smile
  - Reverse Skew
  - Frown
  - Forward Skew
- We use multinomial logit to study the selection into these group.

Smile classification

Month	Smile	Reverse Skew	Frown	Forward skew
1	241 (27%)	592 (65%)	55 (6%)	18 (2%)
2	879 (46%)	768 (41%)	151 (8%)	93 (5%)
3	1212 (65%)	404 (22%)	115 (6%)	121 (7%)
4	1384 (78%)	165 (9%)	46 (3%)	185 (10%)
5	1260 (75%)	120 (7%)	17 (1%)	293 (17%)
6	1009 (69%)	94 (6%)	16 (1%)	335 (23%)
7	657 (62%)	125 (12%)	11 (1%)	275 (26%)
8	429 (59%)	78 (11%)	9 (1%)	217 (30%)
9	316 (60%)	37 (7%)	1 (0%)	172 (33%)
10	241 (58%)	20 (5%)	1 (0%)	152 (37%)
11	200 (57%)	12 (3%)	2 (1%)	135 (39%)
12	152 (50%)	30 (10%)	0 (0%)	119 (40%)
Sum	7980(62%)	2445(19%)	424(3%)	2115(16%)

Multinomial logit regression result

Table: **Multinomial logistic regression result**

Variable	Smile	Reverse Skew	Forward skew
Constant	-0.13	-1.55*	-0.17
Spread	23.78*	22.86*	-29.47**
OptionVolume	0.25***	-0.21***	0.01
Basis	3.69**	-3.05*	10.39***
HP	16.12***	10.08***	1.71
FutureVolume	0.14	0.05	0.06
Monday	-0.07	0.01	0.01
DTM	0.014***	0.011***	0.02***

Reference category: Frown.

Robustness check 2

- We use Multivariate Fractional Polynomials to select the functional forms for implied volatility as a function of moneyness.
- This is an algorithm that chooses the functional form with the best fit.

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- In the first stage, we estimate the relationship between implied volatility and moneyness.
- In the second stage, conditional on a given functional form describing the relationship, we investigate several potential determinants of the β-coefficients.

Conclusion

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Conclusion

- We find a significant correlation between the shape of implied volatility functions and basis and hedging pressure.
- The volatility function tends to be flatter when either basis is low or commercial hedgers are net long.
- The underlying asset distribution exhibits more kurtosis during times of high basis or high hedging pressure.
- We find that the Abdi and Ranaldo (2017) spread estimator and option trading volume are also important determinants of the shape of the implied volatility function.

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The Black-Scholes Formula

Implied volatilities are computed from data using the Black-Scholes formula for call options given by

$$c = e^{-rT}\left(F_0 N(d_1) - KN(d_2)\right) \tag{3}$$

where N refers to the c.d.f . of a standard Normal distribution and

$$d_1 = \frac{\ln\left(\frac{F_0}{K}\right) + \frac{\sigma^2}{2}T}{\sigma\sqrt{T}} \tag{4}$$

$$d_2 = d_1 - \sigma\sqrt{T} \tag{5}$$