Hedging rainfall exposure through hybrid financial instruments

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Outline

Climatic changes and meteorological risk hedging I

Recent international initiatives aim at reducing carbon emissions to limit global warming. (Paris Agreement - November 2016)

Risks connected to climatic changes are unpredictable in their consequences and expose a large number of agents to potential huge losses.

Even if it is estimated that 80% of world industries (agriculture, construction sector and resort locations activities) are affected by climate on a daily bases, financial markets have, so far, not cared much about this topic, assuming this task could be better managed by insurance companies.

An exception is represented by the issuance of catastrophe bonds, that embed the option not to repay the principal in case an extreme event occurs before their expiration.

Climatic changes and meteorological risk hedging II

Since 1997, a large portion of meteorological risk can be hedged exploiting a number of weather derivatives that are actively traded on the Chicago Mercantile Exchange (CME - `www.cmegroup.com/trading/weather`).

With regards to temperatures, such contracts are

- heating degree day (HDD) in US and Europe,
- cooling degree day (CDD) in US, and
- cumulative average temperature (CAT) in Europe.

On the precipitation side, unfortunately, at least to the best of our knowledge, no actively traded derivatives are available on regulated markets.

Temperature based derivatives traded on the CME consider observations taken from a number of benchmark cities: in the US, these cities are

- Atlanta
- Chicago
- Cincinnati
- Dallas
- Las Vegas
- Minneapolis
- New York
- Sacramento

Climatic changes and meteorological risk hedging III

while in Europe only temperatures of Amsterdam and London are used.

It results clear that the European benchmark is unable to offer effective hedges against behaviors of temperature in cities as, for instance, Moscow or Rome.

Meteorological risk can be divided in two instances:

- a **low-to-medium** risk scenario; here, Weather Derivatives (WD) are suited as they can effectively protect against minor diversions from the normal behavior of the underlying variable;
- a **high-to-extreme** risk scenario that occurs, instead, when some extreme meteorological events (for instance a tornado or a severe hailstorm) batter a limited area. In these cases, standard insurance contracts are preferable.

Climatic changes and meteorological risk hedging IV

This comparison leads to the main idea behind our research project. WDs are assumed to be **liquid contracts** as they are traded on regulated markets. Their prices should be, for this reason, closer to the fair value of the traded risk.

Geographically speaking, their hedging ability, in most instances, is poor as data are sampled from a limited number of places.

On the other hand, insurance contracts could cover a large portion of meteorological risk but, being highly **illiquid**, are prone to severe mis-pricing.

In fact, insurance companies collect highly detailed meteorological data from a huge number of locations so their forecasts are very accurate.

This being said, the topic of this talk is to propose an hybrid financial instrument that benefits from the pros of both WDs and insurance contracts while trying to avoid their cons.

Literature review I

A number of books and papers have addressed the valuation of weather derivatives. For the sake of brevity, a very limited list follows:

- Benth, F. E., and Saltyte-Benth, J. (2013). Modeling and Pricing in Financial Markets for Weather Derivatives. Advanced Series on Statistical Science & Applied Probability, Volume 17. World Scientific.
- Benth, F. E., and Saltyte-Benth, J. (2005). Stochastic modelling of temperature variations with a view towards weather derivatives. Applied Mathematical Finance 12(1), 53-85
- Benth, F. E., and Saltyte-Benth, J. (2011). Weather derivatives and stochastic modelling of temperature. International Journal of Stochastic Analysis 2011, 576791
- Alaton, P., Djehiche, B., and Stillberger, D. (2002). On modelling and pricing weather derivatives. Applied Mathematical Finance 9(1), 1-20

Literature review II

- Jewson, S., Brix, A., and Ziehmann, C. (2005). Weather Derivative Valuation. Cambridge University Press
- Schiller, F., Seidler, G., and Wimmer, M. (2012). Temperature models for pricing weather derivatives. Quantitative Finance 12(3), 489-500
- Hess, M. (2018) Pricing temperature serivatives under weather forecasts. International Journal of Theoretical and Applied Finance 21(5), 1850031
- Stefani, S., Moretto, E., Parravicini, M., Cambiaghi, S., Sonubi, A., Kutrolli, G., and Tulli, V. (2018). Managing adverse temperature conditions through hybrid financial instruments. Journal of Energy Markets 11(3), 25-41

Weather derivatives pricing I

The main issue when pricing WDs is that the underlying asset (temperature, rainfall) is not traded on a regular market.

Many of the standard assumptions of mainstream derivative pricing theory cannot, then, be exploited.

Further, temperature historical data show an obvious annual cyclical behavior with a possible long-term non-linear drift component, due to climatic changes.

Rainfall is prone to periodical pattern as well. Usually, 'dry' periods are followed by 'rainy' month.

The quantity of rain per square meter per hour is another variable that lately seems to be steadily increasing as a result of greater quantities of energy captured by the atmosphere.

It results that it is not easy to identify a stochastic model capable of capturing such a full range of features.

Weather derivatives pricing II

A widely applied approach is the **burn analysis**, based on the analysis of meteorological time series that are projected, exploiting some simulation technique, in the near future.

Alternatively, a theoretical model can be used. A recent example comes from Hess (2018): in his article the author exploits a mean-reverting process that incorporates seasonality.

On top of this, theoretical models need to be calibrated. Unlike the stock market, where historical time series are, at best, misleading, because of the well known volatility underestimation problem, temperature dynamics are, at least to some extent, better captured exploiting collected data.

In the temperature case, it is possible to consider a process

$$T(t) = \mu(t) + \epsilon(t)$$

Weather derivatives pricing III

that contains a mean component $\mu(t)$, which models the trend, and a residual component $\epsilon(t)$ which tries to explain fluctuations around $\mu(t)$ over time.

The mean component contains two terms:

- a deterministic function of time

$$\theta(t) = \sum_{u=0}^{v} a_i t^i + a_{v+1} \cos\left(\frac{2\pi t}{365}\right) + a_{v+2} \sin\left(\frac{2\pi t}{365}\right),$$

  embedding a truncated Fourier series and
- an autoregressive process AR (p) with parameters λ_i, $i = 1, ..., p$

Weather derivatives pricing IV

$$\mu(t) = \theta(t) + \sum_{i=1}^{p} \lambda_i \left(T\left(t-i\right) - \theta\left(t-i\right)\right)$$

The residual component, instead, is the product

$$\epsilon(t) = \sigma(t)\gamma(t)$$

where

$$\sigma^2(t) = b_1 + \sum_{u=1}^{w} \left[b_{2u} \cos\left(\frac{2\pi t}{365}\right) + b_{2u+1} \sin\left(\frac{2\pi t}{365}\right)\right]$$

is, again, a truncated Fourier series while $\gamma(t)$ represents a sequence of i.i.d. standard Gaussian random process.

Rainfall can be modeled introducing a binary random variable

$$\eta(t) = \begin{cases} 1 & \text{if event } A \text{ occurs} \\ 0 & \text{otherwise} \end{cases},$$

Weather derivatives pricing V

where event $A(t)$ denotes if it will rain in day t.

The probability of this event obviously seasonally depends on time

$$P[\eta(t) = 1] = \theta(t)$$

The quantity of rain at some day is then

$$p(t) = \eta(t)\sigma(t)\gamma(t)$$

where $\sigma(t)$ and $\gamma(t)$ are modeled as seen before.

Quantity $\theta(t)$ can be expressed in terms of an exponential random variable

$$\theta(t, x, \xi) = \begin{cases} \xi e^{-\xi x} & \text{if} \quad x \geq 0 \\ 0 & \text{if} \quad x < 0 \end{cases}$$

where $\xi > 0$ is a parameter such that $E[\theta(t, x, \xi)] = 1/\xi$, or by exploiting more complex random variables.

Weather derivatives pricing VI

The simplest temperature derivative contracts are HDD and CDD.

Denoting with $T(t)$ the average (i.e. arithmetic mean between the minimum and maximum daily temperatures) temperature measured at some time t and fixing some reference temperature K (usually $K = 18^\circ$), HDD(t) and CDD(t) pay-offs in t are those of a put and call options:

$$\mathrm{HDD}(t) = \max\left(K - T(t); 0\right) \qquad \mathrm{CDD}(t) = \max\left(T(t) - K; 0\right) \tag{1}$$

HDD and CDD indexes between two dates τ_1 and $\tau_2 > \tau_1$ are the following sums

$$\mathrm{HDD}\left(\tau_1; \tau_2\right) = \sum_{t=\tau_1}^{\tau_2} \mathrm{HDD}(t) \qquad \mathrm{CDD}\left(\tau_1; \tau_2\right) = \sum_{t=\tau_1}^{\tau_2} \mathrm{CDD}(t)$$

The amount paid at maturity by either an HDD or a CDD derivative covering interval $[\tau_1; \tau_2]$ in τ_2 is the HDD or CDD index multiplied by a fixed amount κ (tick size).

Weather derivatives pricing VII

As said in the Introduction, no rainfall derivatives are currently traded on a market.

A way of shaping such derivatives is by means of cumulative precipitation (CP) in a given period $[\tau_1; \tau_2]$ expressed as

$$\mathrm{CP}\left(\tau_1; \tau_2\right) = \sum_{t=\tau_1}^{\tau_2} p(t)$$

Given some threshold K, a derivative covering the risk of a dry period will have a pay-off similar to the one of a put option

$$\kappa \max\left[K - \mathrm{CP}\left(\tau_1; \tau_2\right); 0\right]$$

while the opposite case hedges the risk of severe rainfall

$$\kappa \max\left[\mathrm{CP}\left(\tau_1; \tau_2\right) - K; 0\right],$$

resembling the pay-off of a call option.

Risk measures: a brief recap I

Let $\mathbf{X}$ be a random variable (rv) depicting the future outcome in $T > 0$ of either an asset, a bond, a derivative contract or, more in general, a portfolio.

In the Artzner, Delbaen, Eber, and Heath (ADEH - 1998) framework, rvs belong to a sample space Ω and are assumed to be divided, by a risk manager, in either acceptable or not acceptable risks.

Let $\mathcal{A}$ (resp. $\mathcal{A}^C = \Omega - \mathcal{A}$) the set containing all acceptable risks. These sets must obey some 'common sense' assumptions

If $\mathbf{Y} \in \mathcal{A}^C$, a risk measure is some function $\rho : \Omega \to \mathbb{R}$ defined as

$$\inf_{\alpha} \{\alpha v : (\mathbf{Y} + \alpha) \in \mathcal{A}\}$$

where α is a positive amount of money while v is the risk-less discount factor in $[0; T]$.

Risk measures: a brief recap II

A risk measure is the smallest amount of money added to a rv that transforms the original, unacceptable, risk into an acceptable one.

Let

$$F_{\mathbf{X}}(x) = P[\mathbf{X} \leq x]$$

the distribution function for rv $\mathbf{X}$ and assume that β, $0 < \beta < 1$, is a probability level.

Two of the the most common and applied risk measures are:

- the *Value-at-Risk* (VaR) defined as

$$\mathrm{VaR}_{\beta}(\mathbf{X}) = -\inf_{z}\{F_{\mathbf{X}}(z) > \beta\},$$

- and the *Expected Shorfall* (ES), that is

$$\mathrm{ES}_{\beta}(\mathbf{X}) = -\mathbb{E}[\mathbf{X}|\mathbf{X} \leq -\mathrm{VaR}_{\beta}(\mathbf{X})]$$

Risk measures: a brief recap III

both these risk measures are, unlike the variance, tail measures as they focus on the behavior of the worst outcomes of a risk.

ADEH's work determines a fundamental result: some measures are unable to capture in full portfolio risk.

VaR, for instance is not a sub-additive measure. Let $\mathbf{X}$ and $\mathbf{Y}$ be two risks. It can be shown that

$$\mathrm{VaR}_{\beta}\left(\mathbf{X}\right) + \mathrm{VaR}_{\beta}\left(\mathbf{Y}\right) \leq \mathrm{VaR}_{\beta}\left(\mathbf{X} + \mathbf{Y}\right). \tag{2}$$

This VaR's feature defies the notion of diversification: when analyzing more than a risk, VaR suggests **not to** hedge by means of a portfolio of rvs but to invest all the money into a single asset. Otherwise said, the rhs of (2) reports that risks $\mathbf{X}$ and $\mathbf{Y}$, measured separately, are less risky than the same risks measured as a portfolio.

Risk measures: a brief recap IV

ES, instead, is sub-additive:

$$\text{ES}_{\beta}\left(\mathbf{X}\right)+\text{ES}_{\beta}\left(\mathbf{Y}\right)\leq\text{ES}_{\beta}\left(\mathbf{X}+\mathbf{Y}\right).$$

This will be a fundamental feature for WD contracts based on this risk measure because it allows to properly measure the overall impact of a portfolio of risks.

Bridging the gap between WDs and insurance contracts

This work proposes to link at least some positive features of weather derivatives and insurance contracts avoiding, as best as possible (inefficiencies and) mis-pricing due to the standard actuary approach.

The idea is to apply risk measures and exploit historical data to obtain a reasonable price for an insurance contract covering meteorological events that exceed some threshold.

Threshold could, of course, be adapted to specific issues.

An amusement park manager or the owner of a ski area hotel have different concerns with respect to, for example, a wine-maker, a farmer or some gas companies.

Dataset and preliminary numerical analysis I

Preliminary results have been obtained considering a dataset based on daily maximum and minimum temperatures (in Celsius degree - ($^\circ C$)) collected by the weather station Molin Bianco in Arezzo, Tuscany (WMO ID: 16172, latitude: $43^\circ 27' 34.81''$N, longitude: $11^\circ 50' 44.5''$E; elevation: 248 meters above sea level) over the timespan ranging from 1951 till 2016.

The sample, consisting of 24,107 observations, shows an absolute average maximum of 34° recorded on June 18, 1990, when the maximum temperature was 39° and the minimum 29° and an absolute average minimum of -8.10° registered on October 30, 1993, when the maximum temperature was -1.2° and the minimum temperature was -15°.

The average temperature of the whole sample is 13.9°.

The levels of skewness and kurtosis show that the distribution of temperature is symmetric but not normal.

Dataset and preliminary numerical analysis II

Table : Descriptive Statistics - daily average temperature in Arezzo: 1951-2016

	Mean	Var	Std Dev	Min	Max	Skew.	Kurt.
Temp. ($^\circ C$)	13.869	50.455	7.103	-8	34	0.019	2.105

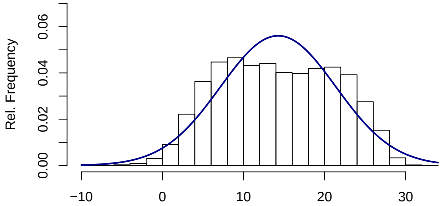

Figure : Histogram of the daily average temperature of Arezzo (1951-2016)

Dataset and preliminary numerical analysis III

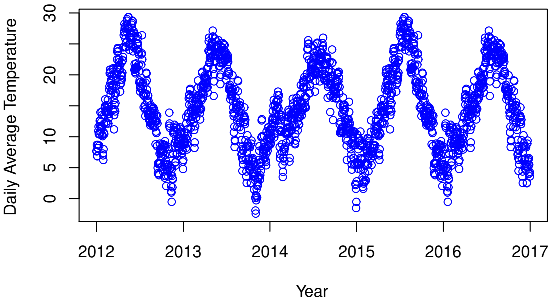

Figure : Time series of daily average temperatures of Arezzo - years 2012 to 2016

Dataset and preliminary numerical analysis IV

As an initial threshold, the tenth $K_1 = 2.43^\circ$ and ninetieth $K_2 = 12.4^\circ$ percentiles of daily temperatures have been applied into (1).

In terms of risk measures, this is like saying that only the coldest 10% and the warmest 10% of daily temperatures are accounted into HDD and CDD indexes.

This is the case as farmers, for instance, might suffer loss in case of very rigid or extremely hot temperatures.

Mimicking Expected Shortfall, the second set of thresholds is obtained computing the sample average of daily temperatures that exceed the Value at Risk threshold.

The hypothesis is that, even if the number of hybrid contracts is not large enough to ensure a predictable distribution, sub-additivity of ES kicks in, allowing to cover properly the risk.

Dataset and preliminary numerical analysis V

Table : Computed thresholds for HDD and CDD

VaR 0.05	VaR 0.1	VaR 0.9	VaR 0.95
0.75	2.43	12.4	13.7
Threshold 1	Threshold 2	Threshold 2	Threshold 1
HDD	HDD	CDD	CDD
ES 0.05	ES 0.1	ES 0.9	ES 0.95
−1.32	0.16	16.42	16.77
Threshold 1	Threshold 2	Threshold 2	Threshold 1
HDD	HDD	CDD	CDD

Dataset and preliminary numerical analysis VI

The final stage is to define a price for a contract (HDD in the table below). Following the common practice, loadings are added to the financial value obtained through burn analysis (Benth, Saltyte-Benth - 2013).

Financial derivative values have been determined exploiting data from January months from 1951 to 2016 and assuming a tick size of $\gamma = 20$ Euro. (upper part of the last table)

These prices have been (partially) tested considering an insurance contract that covers January 2017 and determining the real pay-off paid by this contingent claim. (lower part of the last table)

As for any rv ES is always less (for a left-tail risk) or greater (for a right-tail risk) than VaR, hybrid contracts based on Expected Shortfall have a smaller price and repay a smaller amount to their owners when compared to those based on Value at Risk.

Dataset and preliminary numerical analysis VII

Table : Pricing hybrid temperature derivatives - preliminaty results (Figures in Euro - tick size $\gamma = 20$ Euro)

January (1951-2016)	HDD - thr. 2.43°C	HDD - thr. 0.16°C
Derivative value	139.58	47.74
Risk loading	34.74	2.55
Final price	174.32	50.29
January 2017	**VaR**	**ES**
Insurance price	174.32	50.29
Payoff	451.60	100.80
Profit/Loss	277.28	50.51

Tick sizes as well as time spans can, of course, be chosen so to match the specific needs of the agent seeking risk coverage.

Dataset and preliminary numerical analysis VIII

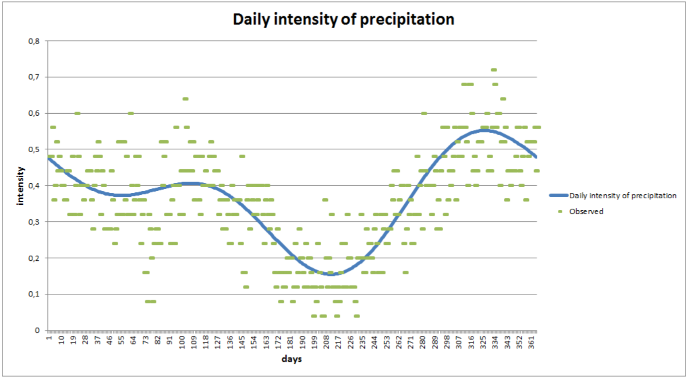

Figure : Rainfall average quantity on a daily basis - Arezzo area - years 1951 to 2016

Dataset and preliminary numerical analysis IX

The graph above shows the historical simple mean precipitations day by day for the Arezzo area under scrutiny.

In order to protect, for instance, against the risk of either a too rainy a or too dry summer, an agent can choose a specific threshold K and benefit of positive pay-offs in case of adverse, even if not extreme, meteorological events.

On the other side, the issuer of this derivative can apply standard pricing valuation techniques and hedge against losses applying the appropriate measure of risk.

Conclusions I

- meteorological risk has dramatically worsened by climatic changes
- the most common tools for managing this risk seem to be only partially helpful as they are either not flexible enough to capture peculiarities of a limited area or far from being priced in an efficient way
- risk measure theory suggests an approach that promises to be interesting; sub-additivity of the Expected Shortfall allows insurance companies or insured subjects counterparts to properly handle the risk they deal with
- goodness of this approach depends on the quality and amplitude of available datasets
- the initial numerical results seem promising; pricing of hybrid contracts (based on temperature and rainfall) can be done with a limited effort

Conclusions II

THANKS FOR YOUR ATTENTION!

To conclude...

You all are kindly invited to the following event:

WHAT: Energy Finance Italia 4th workshop

WHEN: February 4-5th, 2019

WHERE: University of Milano-Bicocca (Italy)

Are you interested (you should...!)?

Will you join us?

More info at

www.ief.unimib.it